MATHEMATICAL MODELING AND SYSTEM ANALYSIS OF CATTLE FEEDLOT RUNOFF

by 45

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B. S., National Taiwan University, 1965

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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1969

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INTRODUCTION

As the water pollution problem has become more acute, interest has been generated in the extent of pollution provided by farm animal water in surface runoff. The necessity of farm animal wastes treatment arises under the radical changes of cattle industry within the past decade. The water pollution aspect of animal wastes consists of fish-kill and disease infection. For instance, Smith(2) reported that of 27 fish kills in Kansas during 1964, fifteen were believed to be caused primarily by runoff from commercial livestock feedlot operations. According to Hull(3), 56 diseases might be transmitted from cattle to man.

The control of water pollution by feedlot runoff can not be accomplished effectively without an appropriate understanding of the system. It is the job of the biochemist, chemist, sanitary engineer, chemical engineer, agricultural engineer, and specialists in many other fields to work out the approved scheme of handling this problem.

A study of the water pollution potential of cattle feedlot runoff has been carried out by Miner(1). In his report, both qualitative and quantitative analyses of the samples from a feedlot system have been made extensively. However, since an independent input variable (rainfall, for example) does not generally remain at a steady state value and since the runoff system is an extremely transient one, an unsteady state dynamic model is needed. Furthermore, the development of a general identification scheme of the pollution dynamics may also be desired. In this study, the

emphasis will be on obtaining a mathematical relation which relates the important dependent variables of the system to the important independent input variables.

Water Pollution (12)

All water comes in the form of precipitation. It is evaporated from the ocean, condenses to form clouds, and precipitates over land. As the water falls in the form of rain or snow or sleet or hail, it picks up the dust and dirt in the air. Naturally, the first water that falls picks up the greatest concentration of contaminants. After a short period of fall, the precipitation is relatively free of impurities. When the water hits the ground, a portion of it sinks into the ground. The water running across the surface of the ground has been designated surface water. It picks up a variety of substances as it flows back to the ocean, such as microorganisms, organic matter, and minerals.

Surface water collects in areas forming lakes and ponds, and being rich in nutrients it becomes a medium for the growth of all types of microorganisms. All forms of microbial life are found in surface water. The types and numbers of microorganisms are a direct reflection of the conditions in the water. If the water is free of minerals, little, if any, biological life will be found. As more organic matter and minerals find their way into the surface water, bacteria, algae, and protozoa grow. Fairly pure water supports a small total number of microorganisms but a relatively large number of different species. As more contaminants enter the water, the total number of microorganisms increases,

while the number of species decreases. Water high in inorganics shows excellent algae growth, while waters polluted with organics show predominantly bacteria growths.

The bacteria are the keys to the normal biological cycle. It is the role of the bacteria to convert the soluble organic matter into bacteria cells and inorganic elements. The inorganic elements are taken by the algae and converted into algae cells. The newly formed bacteria and algae become food for the protozoa, rotifers, and crustaceans. The animal forms and some of the large algae and bacteria become food for the minnows and tiny fish. The small fish become food for the large fish which become food for man. Man discharges his wastes back into the stream where the bacteria metabolize the organic matter and complete the so-called normal biological cycle, as shown in Figure 1.

A high mountain stream shows little biological life because of the lack of nutrients in the water but a river flowing through a rich agricultural area will teem with biological life. The demand for oxygen in a stream is a direct result of the aerobic metabolism of the microorganisms in the stream. Normally, the metabolic activity in streams is limited by the lack of nutrients. The low organic level permits only a few bacteria and fungi to grow. These in turn limit the growth of the animals in the stream. The sudden introduction of organic wastes increases the food concentration and allows the bacteria and fungi to speed up their rate of growth. The increased growth results in a proportional decrease in oxygen concentration in the stream. If the organic load is sufficiently high, the entire oxygen resources

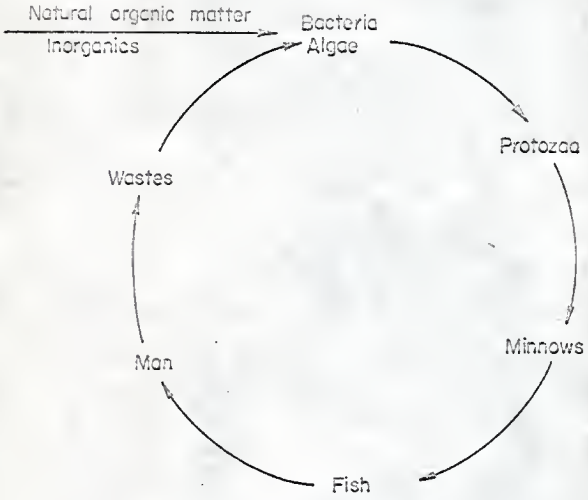


Fig. 1. Natural biological cycle in streams (12).

are depleted and the stream becomes anaerobic, with resultant odors and black colors.

In liquid wastes, dissolved oxygen is the factor that determines whether the biological changes are brought about by aerobic or by anaerobic organisms. The former use free oxygen for oxidation of organic and inorganic matter and produce innocuous end products, whereas the latter brings about such oxidations through the reduction of certain inorganic salts such as sulfates, and the end products are often very obnoxious. Even man is unable to live close to the polluted river.

If nuisance conditions are to be avoided in a stream, or if certain types of fish are to be maintained, it is essential that the dissolved oxygen level in the water does not drop below certain values.

The water flowed over a feedlot area is heavily polluted, because the manure is a rich organic material. Immediately following rainfall, septic conditions, high BOD, high ammonia concentrations, and high bacterial counts were recorded at the time of the observed fishkills. For example, Henderson(20) reported in 1962 that, under appropriate conditions, the BOD from agricultural runoff makes the sewage and industrial waste load of the area of Potomac River watershed above Washington, D. C. seem insignificant.

Chemical Oxygen Demand and Biochemical Oxygen Demand (12, 19)

Adequate dissolved oxygen (abbreviated DO) is necessary for the life of fish and other aquatic organisms. The DO concentra-

tion may also be associated with corrosivity of water, photosynthetic activity, and septicity.

In dealing with the pollution potential of a water, a single quantitative enumeration of the general type of microorganisms such as bacteria, algae, fungi, protozoa, and rotifers is merely a qualitative measure of the stream condition. It has a limited value in itself.

The Biochemical Oxygen Demand Test (BOD Test) measures the organic strength of sewage water. It is essentially the measurement of the oxygen needed to stabilize the organic matter in contaminated water, or the amount of oxygen required by bacteria while stabilizing decomposable organic matter under aerobic conditions. The 5-day BOD test comes from England and was evolved from the fact that in England none of the streams had more than a 5-day flow period, and thus, the 5-day BOD is the maximum oxygen demand which would be exerted in any stream in England. The B.O.D. test transported to the United States lost most of its original significance since many streams have more than 5 days flow to reach the ocean.

The prime factor in the B.O.D. test is the presence of microorganisms in the sewage. The test is essentially a bioassay procedure involving the measurement of oxygen consumed by living organisms (mainly bacteria) while utilizing the organic matter present in a waste, under conditions as similar as possible to those that occur in nature. The B.O.D. test is, therefore, widely used to determine the pollutional strength of domestic and industrial wastes in terms of the oxygen that they will require if

discharged into natural watercourses in which aerobic conditions exist.

The Chemical Oxygen Demand (C.O.D.) determination is a measure of the oxygen equivalent to the portion of the organic matter in a sample that is susceptible to oxidation by a "strong" chemical oxidant, under acid conditions. During the determination of C.O.D., organic matter is converted to carbon dioxide and water regardless of the biological assimilability of the substances. It is, therefore, a rapidly measureable parameter for stream and industrial-wastes studies and control of waste treatment plants. In the absence of a catalyst, however, the method fails to include some organic compounds (such as acetic acid), which are biologically available to the stream organisms, while including some biologic compounds (such as cellulose), which are not a part of the immediate biochemical load on the oxygen assets of the receiving stream. As a result, C.O.D. values are greater than the corresponding B.O.D. values and may be much greater when significant amounts of biologically resistant organic matter is present. Consequently, the chief limitations of the C.O.D. test is its inability to differentiate between biologically oxidizable and biologically inert organic matter. In addition, it does not provide any evidence of the rate at which the biologically active material would be stablized under conditions that exist in nature. In spite of these shortcomings, however, the C.O.D. test is widely used as a means of measuring the pollutional strength of waste water. The reason is that the C.O.D. test has the advantage of speed of measurement. The determination can be made in about 3

hours rather than the 5 days required for measurement of B.O.D. It is, therefore, used as a substitute for the B.O.D. test in many instances. C.O.D. data can often be interpreted in terms of B.O.D. values after sufficient experience has been accumulated to establish reliable correlation factors. For example, Miner(1) reported that a factor of about 9.5 was observed for feedlot runoff water.

In this study, accordingly, the concentration of Chemical Oxygen Demand is used as the measure of pollution strength, and the basis for system analysis of feedlot runoff.

General Description of the Feedlot System (1)

Two experimental cattle feedlots were built near the Kansas State University campus. One was surfaced entirely with concrete, the other had concrete only around the feed bunks. Each lot was 92 by 24 feet (0.5 acre) with a constant slope in the lengthwise direction of two per cent. Six-inch concrete curbs prevented entrance of runoff from other areas and restricted runoff to one outlet point in each lot.

Twelve part-circle irrigation sprinklers were placed around the periphery of the lots in six locations to simulate rainfall. Provisions were made to allow operation of one or more sprinklers at each location. Nozzles of ¼- and 7/32- inch diameter were used to provide flexibility of rainfall application rates. The system produced rainfall intensities ranging from 0.4 to 2.5 inches per hour. The sprinklers were mounted inside specially cut barrels. These barrels were cut to limit sprinkling to the

lots and collect other water for return to the water source. Twelve rainfall collection cans were distributed in each lot to measure the amount of rainfall applied.

From the discharge point of each lot, runoff dropped into a rectangular box which was the approach to an HS-type flow measuring flume. Specially constructed proportional sampler were located to pass through the flume discharges and transfer a sample to a steel barrel for compositing. Figure 2. shows the layout of the experimental feedlot. The measured samples are expressed in terms of C.O.D.

The experimental data obtained by Miner(1) are used as the source data in this work.

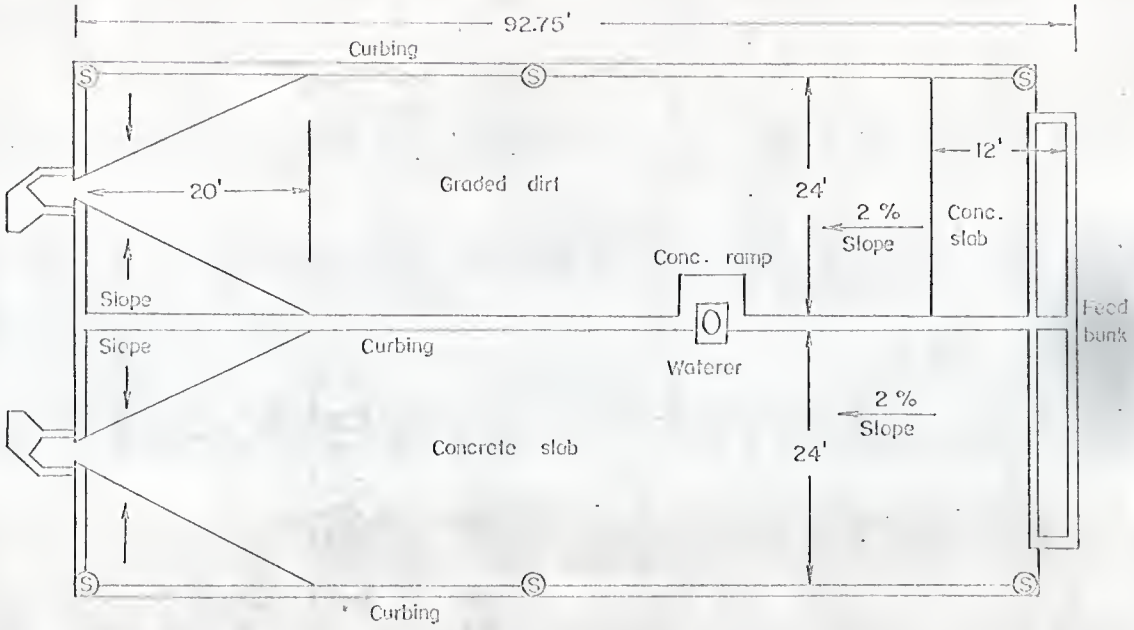


Fig. 2. Layout of experimental feedlots (1).

MATHEMATICAL MODELING OF THE SYSTEM

Models are, by definition, simplifications of the actual processes which they represent. For this reason, models must be neither so simple that the actual process cannot be represented, nor so complex that no insight into the significant characteristics of the system is gained. The study of water pollution due to surface runoff requires consideration of both quantitative and qualitative aspects. Each aspect, however, is controlled by a variety of factors, many of which are hard to evaluate.

The difficulty of describing the feedlot runoff system in a simple mathematical expression comes from the complexity of the system itself. However, a gross behavior prediction of the system can be made by neglecting less important effects and concentrating upon only the significant features of the system.

Models which may represent the feedlot system have been suggested by Miner(1). In his work, however, some simplifications have been made to overcome the mathematical complexity in regard to the nonlinear property of the system. The development of modern computers has made it possible to solve a set of nonlinear differential equations effectively by either analog simulation or through the use of digital numerical integration technique (for example, the Runge-Kutta method of numerical integration). In this work, the dynamic behavior of the system has been modeled and studied by taking into account the nonlinear characteristics of the system.

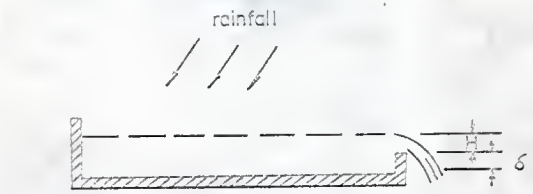
Shallow Basin Model— Hydrological Model

Two types of hydrologic modeling of feedlot runoff system have been suggested by Miner(1). The first one is called the sloping plate model, in which the system is considered as a flat sloping plate, originally dry and clean, to which water is added uniformly from the top. This unsteady state model would yield runoff quantity as a function of rainfall intensity, slope of the plate, and time. However, this simplest model does not represent the physical system satisfactorily as reported by Miner(1). Another type of the model is called the shallow-basin model.

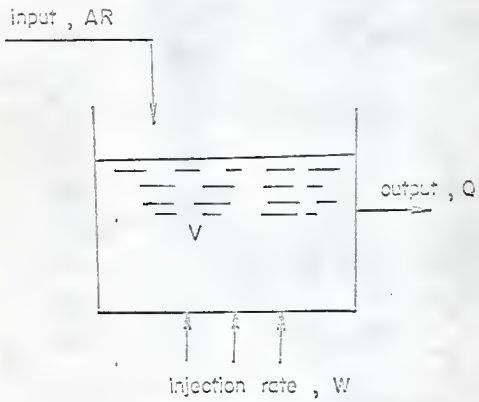
In the second model, the runoff pattern from a feedlot is approximated by considering the feedlot to be a single shallow basin. As a matter of fact, a feedlot surface consists of many small basins formed by the cattle hooves in the soft surface. In this model, discharge is controlled by flow over an imaginary single broad-crested weir at the lower end of the lot. Thus no discharge would take place until the reservoir is filled to the level of the weir crest. Once the runoff starts, the discharge increases until it reaches a steady state flow rate, that is, equal to the rate of rainfall on the lot surface. As reported by Miner, this model appears to represent reasonally well the hydrology of the experimental feedlots. In this study, therefore, this model is employed for representing the hydraulic behavior of the system. The side-view of the model may be represented as Figure 3 (a).

The flow over a broad-crest weir, according to King(4), may be represented by the following expression

$$Q = \lambda L H^{3/2} \qquad (1)$$



(a). Shallow Basin Model.



(b). Stirred Tank with Injection Model.

Fig.3. Model for Feedlot System.

where Q = discharge from feedlot, cfs

λ = weir constant, 2.6 suggested, $ft^{1/2}/sec$

L = weir length, ft

H = water head over the weir crest, ft

Stirred Tank with Injection Model — Mass Transfer Model

To characterize the mass transfer aspect of the system, three models have been proposed by Miner(1), namely, the Stirred Tank model, the Stirred Tank with Injection model, and the Series Stirred Tank with Injection model. The simplest model among them is the Stirred Tank model. In this model the feedlot system is considered as a single completely stirred tank, with an initial concentration of COD, C_o. The runoff concentration of COD with respect to time can thus be represented by the following differential equation

$$V \frac{dC}{dt} = - CQ, \qquad C = C_o \quad \text{at} \quad t = 0 \tag{2}$$

where C = concentration of COD

V = tank volume

Q = out flow rate

t = time.

This over-simplified model, which shows a monotonic decrease of COD concentration with respect to time, does not fit the experimental data satisfactorily.

A modification of the preceding model is called the Stirred Tank with Injection model. According to this model, the system

may be visualized as a single completely mixed tank originally filled with a solution to which both clear water (rainfall) and a feed(manure) are added at constant rates. This model recognizes that not all of the organic matter on the lot surface is soluble immediately but assumes that it enters the solution at a constant rate, W, after the runoff is started. The model is illustrated in Figure 3(b).

Another more complex model is the Series Stirred Tanks with Injection model. This model considers the feedlot system as a connection of two stirred tanks in series, the first of which receives injection. This second tank initially contains no manure but is partially filled with water. The water in the second tank represents that portion of rainfall which lands near the outflow and runs off without having had significant contact with the manure. Needless to say, a more complex model would give a better representation of the real system, however, it introduces additional parameters, which in turn make the analysis of the system's behavior more difficult. According to the recommendation of Miner(1), the Stirred Tank with Injection model is chosen to represent the mass transfer behavior of the system.

The balance of COD based on this model can then be written as

$$V \frac{dC}{dt} = W - CQ , \qquad C = C_o \quad \text{at} \quad t = 0 \tag{3}$$

where W = injection rate, lbs./min.

C = chemical oxygen demand(COD), lbs./cu.ft.

V = tank volume, ft^3

Q = outflow rate, ft^3/min.

The time is measured from the onset of the runoff. In Equation (4), the tank volume V is defined as

$$V = A(H + \delta) \tag{4}$$

where, A = area of feedlot, ft^2

δ = weir height, ft.

H = head over the weir crest, ft.

If the volume change of the water caused by the dissolving organic matter and suspensions is neglected and, furthermore, the water loss from feedlot surface by infiltration and evaporation is ignored, the function of the water head may be represented in a first-order differential equation, i.e.

$$A \frac{dH}{dt} = AR - Q , \qquad H = 0 \quad \text{at} \quad t = 0 \tag{5}$$

where R represents rainfall intensity in ft./min., and the initial time is assigned at the point when the runoff starts.

In equations (2), (3), (4), and (5), the initial concentration C_o, the injection rate W, and the weir height δ appear as three parameters of the system which are to be determined. The equations are nonlinear. They can not be solved analytically. An analog computer may be employed to solve the equations for a set of given values C_o, W, and δ.

The assumptions made in the foregoing mathematical model can be summarized as follows:

(1) The system is assumed to be time-invariant for simplicity; therefore δ can be considered as a constant.

(2) Solid suspensions in solution are neglected in the material balance.

(3) The injection rate W is assumed constant throughout the rainfall period.

(4) Only the mean value of rainfall intensity is considered.

(5) The rough lot surface with pot-holes made by cattle hooves is taken into account by an assumed weir crest δ.

(6) The slope effect of the feedlot is ignored.

(7) The solution in the tank is assumed to be stirred completely so that the concentrations are the same both in the tank and at outflow.

(8) Infiltration and evaporation processes of the system are neglected.

ANALOG SIMULATION AND DETERMINATION OF PARAMETERS

Simulation as a technique for analyzing the behavior of physical systems has a long history, but only in recent years has its use become widespread as a tool for problem-solving. This has come about as part of the rapid development of operations research and systems analysis and associated advances in electronic computers.

The term "computer simulation" means that the differential equations which represent the dynamic characteristics of the system components are solved simultaneously to produce time-varying output voltages which resemble the transient behavior of the real system (7). In a sense, simulation is nothing more than a solution of a set of simultaneous differential equations. The analog computer is now recognized as an important tool in the study of transient behavior and control. Functions of the analog computer consist of both linear and nonlinear operations. The linear operations include summation, integration, and multiplication by a constant. The nonlinear operations include a function generater, multiplier, diode etc. Through the combined use of these operations, a high order linear differential equation or a nonlinear equation can be solved without difficulty. The approach for analog simulation is essentially the same for both nonlinear and linear problem.

The computer simulation method offers the following advantages:

(1) The dynamic response of a system for various values of process parameters can be obtained with little effort once an analog circuit has been constructed. The parameters are usually

changed merely by adjusting potentiometers.

(2) The computer can be operated as a component in an actual physical process.

(3) The computer can be time-scaled so that the response from the computer is very fast compared with the response of the actual system which is being simulated.

The limitation of the use of analog computer to solve a problem depends on the capacity of the computer and the required operators in solving the desired equation or equations.

Simulation of the Feedlot System

The four equations which represent the feedlot runoff system are rewritten here for convenience. They are

$$Q = \lambda L H^{3/2} \tag{6}$$

$$V = A\,(H + \delta) \tag{7}$$

$$A \frac{dH}{dt} = AR - Q, \qquad H = 0 \quad \text{at} \quad t = 0 \tag{8}$$

$$V \frac{dC}{dt} = W - CQ, \qquad C = C_0 \quad \text{at} \quad t = 0 \tag{9}$$

where Q = out-flow rate

δ = weir height

L = weir length

λ = a constant

H = head over weir crest

V = tank volume

A = area of feedlot (cross-section area of tank)

R = rainfall intensity

C = COD concentration

W = injection rate.

These four equations are to be solved simultaneously with the given initial conditions.

To obtain satisfactory results from an analog computer, problem variables should be properly scaled. The scaled quantities are then substituted into the original equations. Let dH/dt and dC/dt be denoted by $\dot{H}$ and $\dot{C}$ respectively, and let

SQ = scale factor for outflow rate, Q;

SR = scale factor for rainfall, R;

SH = scale factor for water head, H;

$S\dot{H}$ = scale factor for dH/dt, or $\dot{H}$;

SC = scale factor for concentration, C;

$S\dot{C}$ = scale factor for dC/dt, or $\dot{C}$;

SV = scale factor for tank volume, V;

SV^{-1} = scale factor for $1/V$, or V^{-1}.

By employing these scale factors, Equations (6), (7), (8), and (9) can be written as,

$$[Q] = K_1[H]^{3/2} \frac{SQ}{(SH)^{3/2}} \tag{10}$$

$$[V] = K_2 \frac{SV}{SH}[H] + K_3(SV) \tag{11}$$

$$[\dot{H}] = \frac{S\dot{H}}{SR}[R] - \frac{1}{K_2}\frac{S\dot{H}}{SQ}[Q] \tag{12}$$

$$[\dot{C}] = K_2 \frac{S\dot{C}}{SV^{-1}}\left[\frac{1}{V}\right] - \frac{S\dot{C}}{(SC)(SQ)(SV^{-1})}\left[\frac{1}{V}\right][C][Q] \tag{13}$$

or

$$[\dot{C}] = \frac{S\dot{C}}{SV^{-1}}\left[\frac{1}{V}\right]\left\{K_4 - \frac{1}{(SC)(SV^{-1})}[C][Q]\right\} \tag{14}$$

where $K_1 = \lambda L$

$K_2 = A$

$K_3 = A\delta$

$K_4 = W$

and [ ] denotes the scaled quantities.

An analog computer circuit diagram for the solution of these four equations is shown in Figure 4, in which four nonlinear operators are employed. The numerical values used are A = 2210 ft.^{2}, L = 24 ft, and λ = 2.6 x 60. The graphical analog solutions of flow rate Q, water head H, and COD concentration C are shown in Figure 5 for a set of given values of C_o, W, and δ. Referring to Figure 5, the outflow rate Q and water head H increase from zero to certain steady state values, while the concentration of COD increases at first then decreases to a steady state value. The time required for flow rate Q, and water head H to reach the steady state condition is relatively shorter than that of the concentration of COD.

The investigation of the effects of changing only one of the three parameters C_o, W, and δ at a time, while keeping the other two constants can be carried out on an analog computer. The effects of varying these parameters on the shape of the output for the same constant input (i.e. rainfall intensity) are illustrated in Figures 6, 7, and 8, in which a rainfall intensity of 0.72 in./hr. has been assumed. The following qualitative tendencies of COD may be noted with regard to Figures 6, 7, and 8.

(1) In Figure 6, the peak concentration increases as C_o becomes larger. However, the concentration reaches the same steady

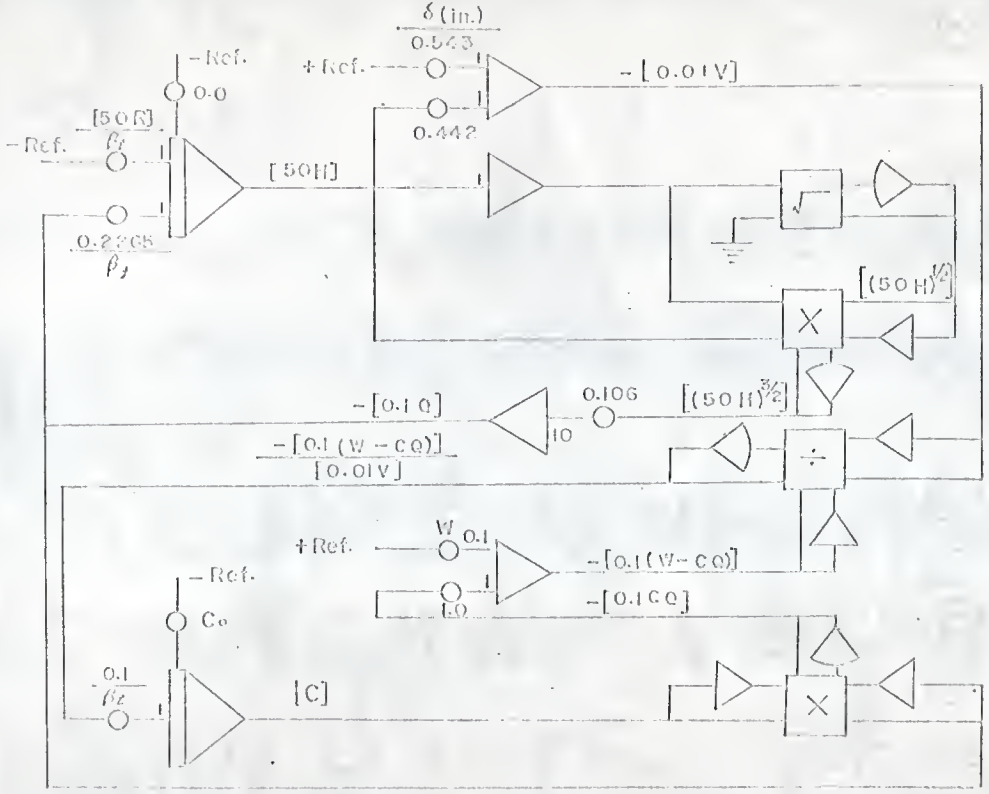


Fig. 4 Analog computer circuit for feedlot runoff system.

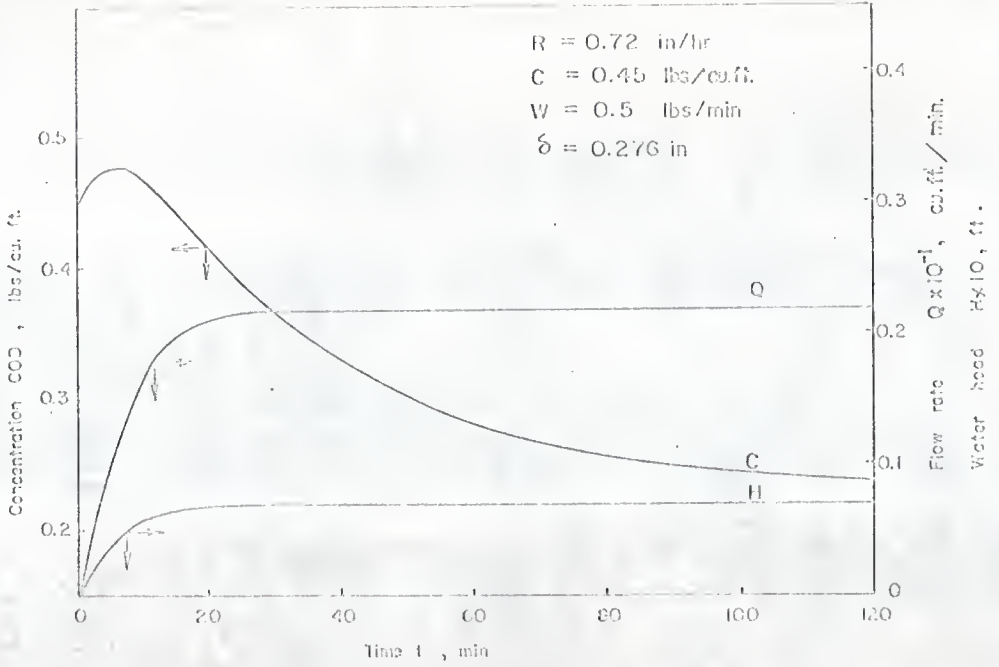


Fig. 5. Analog solution of COD, flow rate, water head vs. time.

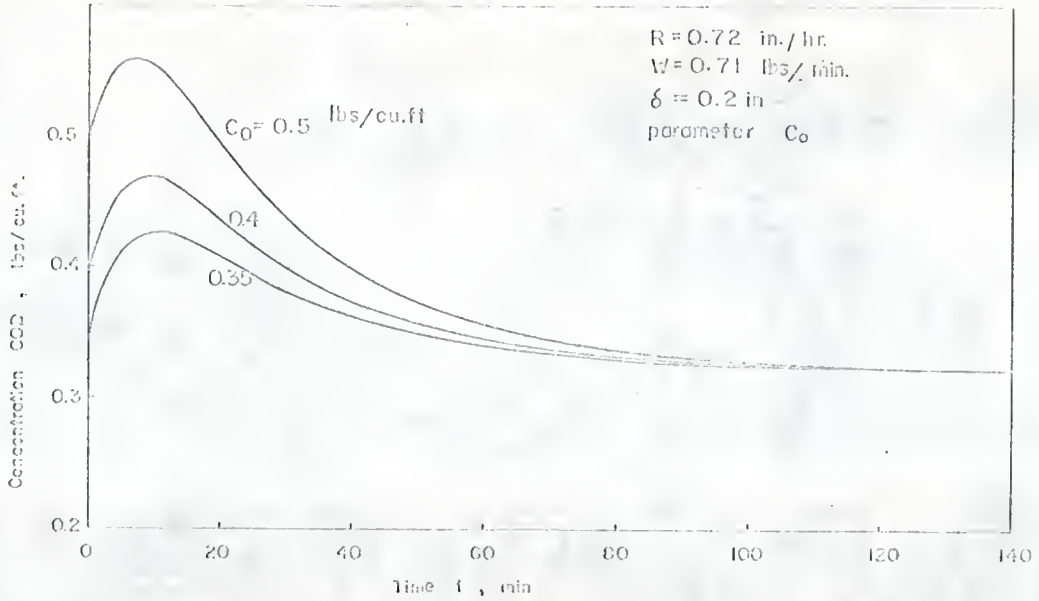


Fig. 6. Effect of initial condition C_o on COD concentration.

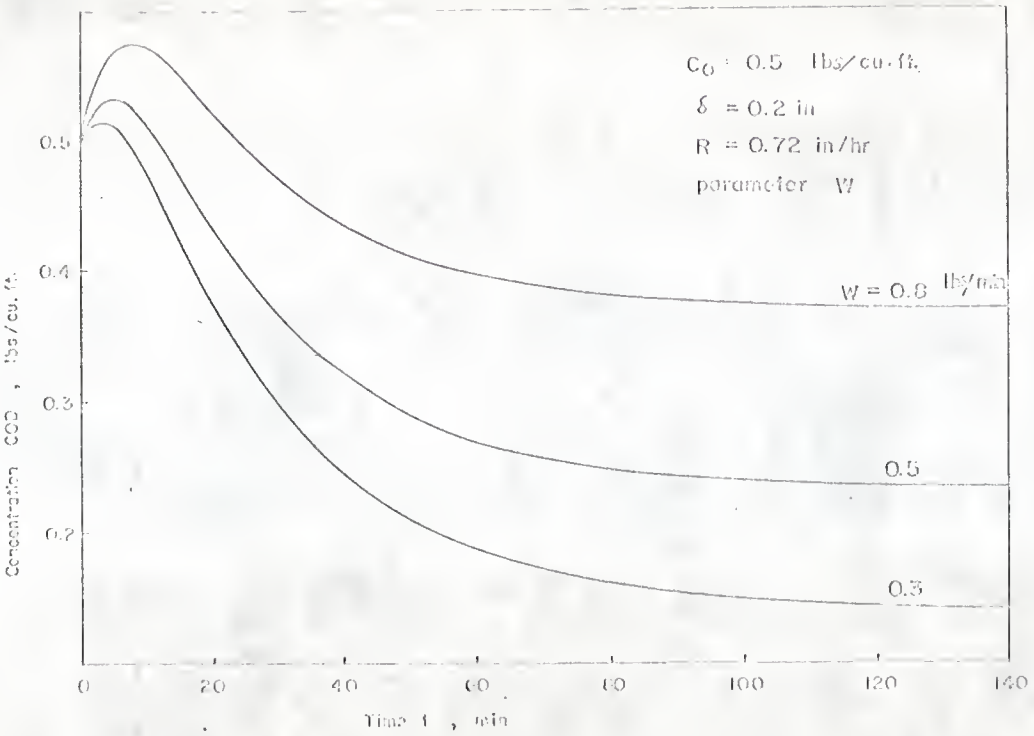


Fig. 7. Effect of injection rate W on COD concentration.

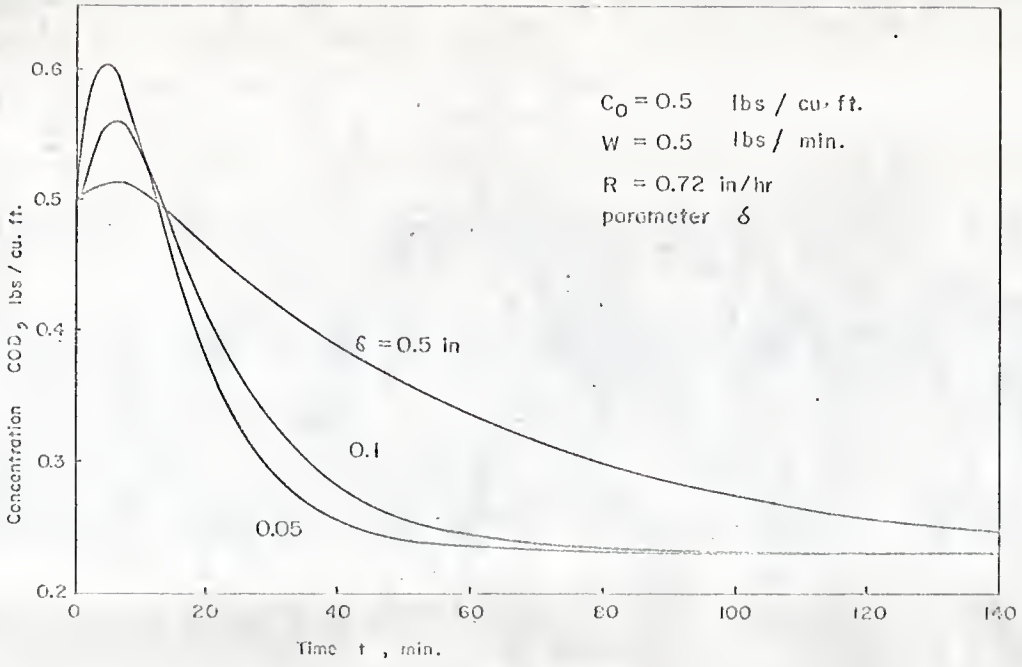


Fig. 8. Effect of weir height δ on COD concentration.

state condition at about the same time, and there is very little decrease in the time of occurrence of the peak concentration as the value of C_o becomes larger.

(2) In Figure 7, the peak concentration of COD, the time to reach the peak concentration, and the steady state concentration increase as W is increased. The time to reach the steady state concentration is not affected by the change of injection rate W.

(3) In Figure 8, the peak concentration decreases a considerable amount as δ is increased, while the time to reach the steady state concentration increases as δ is increased. The time of the occurrence of peak concentration seems to increase slightly for a larger value of δ.

An interpretation(1) of the initial increase of runoff concentration of COD is that it is due to the arrival at the discharge point of water which has traveled the full length of the feedlot. Initial runoff is relatively uncontaminated water which has fallen near the discharge point. Once runoff has been established, the decrease in COD concentration is due to a decrease in contact time between rainfall and the feedlot litter and to a removal of the soluble portion of the litter.

Determination of Parameters of the System

As mentioned previously, one of the advantages of using an analog computer in solving differential equations is that the parameters of a model representing the system under consideration can be represented by potentiometers, and therefore, the effect of changing the parameter can be investigated by merely adjusting

the appropriate pot settings and observing the output curves.

The three parameters contained in the proposed model of the feedlot system, namely the initial concentration C_o, the injection rate W, and the weir height δ are to be determined. This can be accomplished by fitting the analog solution to the experimental data in accord with different rainfall intensity. Twenty sets of experimental data which corresponding to various situations of feedlot surface are available in Miner's dissertation(1). By using the rainfall intensity measured in each experimental run, and the following numerical values

$$A = 2210 \text{ ft}^2$$

$$\lambda = 156 \text{ ft}^{\frac{1}{2}}/\text{min}$$

$$L = 24 \text{ ft},$$

different sets of parameters have been obtained. Simultaneous determination of the values of C, W, and δ for a proper fit COD curve is essentially a matter of trial and error. The qualitative investigations made by changing parameters as shown in Figures 6, 7, and 8 are thus helpful to obtain adequate fitting.

The rainfall intensities and surface conditions together with the parameters obtained by data fitting are shown in Table 1. An examination of these parameters shows that the value of the initial concentration ranges from 1.2 to 0.2, with most values lying around 0.45, while the value of W ranges from 1.2 to 0.1 and δ from 0.55 to 0.1.

Table 1. Parameters determined by fitting the analog solution of COD to experimental data.

Experimental run(a)	Lot condition(b)	Rainfall intensity in./hr.	Initial conc. lbs/ft^3	Injection rate lbs/min	weir height in.
1(S)	SM	0.72	1.200	0.800	0.543
1(U)	SM	0.88	0.400	0.600	0.217
2(S)	SM	0.88	0.550	1.150	0.543
2(U)	SM	0.98	0.375	0.675	0.543
3(S)	D	0.34	0.430	0.195	0.109
3(U)	D	0.53	0.520	0.650	0.135
4(S)	W	0.40	0.900	0.600	0.109
4(U)	W	0.50	0.500	0.550	0.272
5(S)	SM	0.39	0.500	0.350	0.380
5(U)	SM	0.38	0.301	0.250	0.217
6(S)	SM	0.36	0.400	0.320	0.136
6(U)	SM	0.46	0.350	0.350	0.190
7(S)	M	0.33	0.480	0.180	0.244
7(U)	P	0.54	0.380	0.430	0.217
8(S)	M	0.43	0.575	0.400	0.191
8(U)	P	0.59	0.361	0.390	0.136
9(S)	M	0.63	0.392	0.335	0.136
9(U)	P	0.42	0.201	0.160	0.217
10(S)	M	0.58	0.590	0.510	0.109
10(U)	P	0.22	0.285	0.101	0.164

Table 1. (cont'd)

note: (a) Miner, J. R.; dissertation (1), pp. 138 - 142

S - concreted lot

U - nonconcreted lot

(b) SM - lot surface slightly moist before experiment began.

D - lot dry before rainfall started.

W - lot saturated with water before experiment began.

M - manure mounded in the lot one day preceding experiment.

P - manure moved to the edges on the lot and the surface smoothed to provide a flow-way down the center on the day preceding experiment.

Stability of Parameters of The Porposed Nonlinear Model

If the parameters of a mathematical model of a physical system have values ranging from a positive quantity to a negative quantity for various inputs, and the mode of variation of these parameters is erratic, then the parameters may be said to be unstable. Instability of parameters may indicate that the system may have been inadequately described by the mathematical model used (21).

The proposed model for the feedlot runoff system has three parameters, namely C_o, W, and δ. These parameters are determined from fitting analog solutions to the experimental data. The method of analog solution has been discussed in the preceding section. The parameter W obtained for various experimental runs is found to be stable and ranges from 1.2 to 0.1.

Intuitively, the injection rate W may be expected to be a function of (1) the solubility of the manure, (2) the area of the manure water contact surface, (3) the water velocity over the manure area, and (4) the disturbance caused by the down-fall of rain water. The last two assumptions suggest that there may be some correlation between rainfall intensity and injection rate W, since heavier rainfall intensity will increase the velocity over the manure area and cause much more disturbance. A simple plot of W versus rainfall intersity R as shown in Figure 9 reveals the fact that these two variables do have very approximately, a linear relationship.

Two types of relationship have been proposed. The first one considers that the relation between W and rainfall intensity R can be represented in the expression.

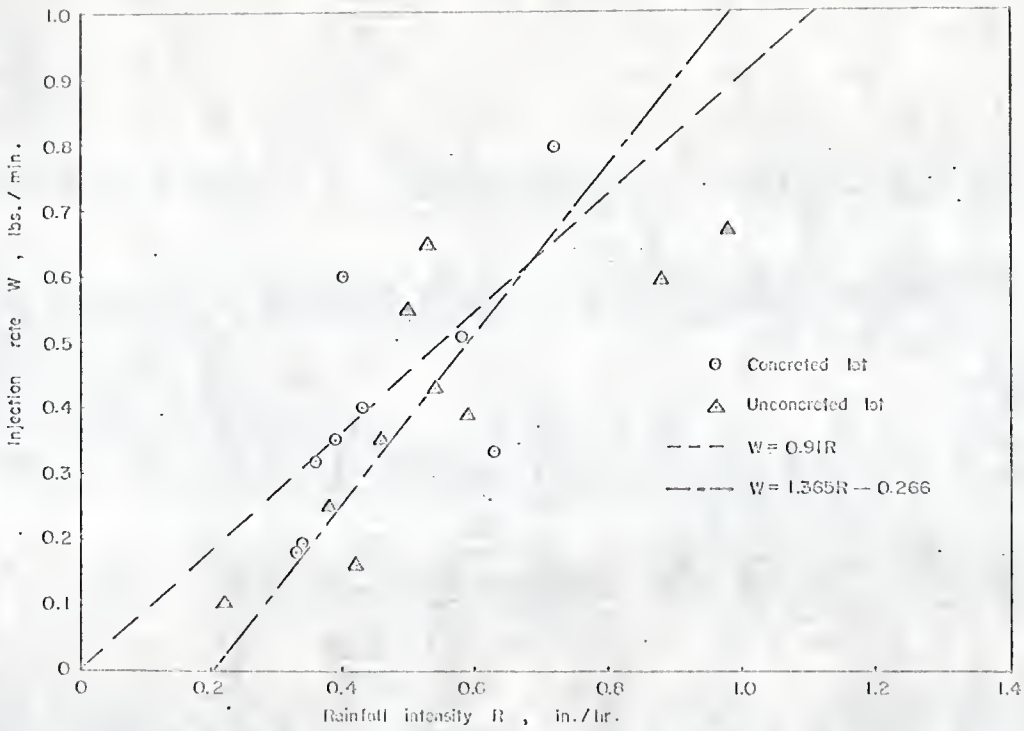


Fig. 9. Injection rate vs. rainfall intensity.

$$W = K_1 R \tag{19}$$

where W = injection rate

R = rainfall intensity

K_1 = a constant.

This expression implies the assumption that, if there is no rainfall, then no injection would occur.

The second one assumes that the relationship is linear with an interception, i.e.

$$W = K_2 R + K_3 \tag{20}$$

where K_2 and K_3 are constants. This expression does not satisfy the assumption that at zero rainfall intensity given by $W = 0$, although it gives the better fitting of the data. By setting $W = 0$ in Equation (20) we obtain,

$$R_c = -\frac{K_3}{K_2} \text{ . (in/hr.)} \tag{21}$$

It may be interpreted physically, that the rainfall intensity has to exceed a certain value R_c to cause injection of organic matter; or, equivalently, no runoff would occur for a rainfall intensity less than this value.

The values of K_1, K_2, and K_3 are thus obtained for the following three different cases.

(1) by considering both concreted and nonconcreted lot together,

(2) by considering only the concreted lot,

(3) by considering only the nonconcreted lot.

The best fitting lines based on the conventional least squares criterion are shown in Figures 9, 10, and 11. For the first case we obtain

$$W = 0.91\ R \tag{22}$$

and

$$W = 1.365\ R - 0.266 \tag{23}$$

Where the units of W and R are in lbs./min. and in./hr. respectively. For the second and third cases, we obtain

$$W = 1.0\ R\ , \text{ and} \tag{24}$$

$$W = 1.44\ R - 0.164 \tag{25}$$

for the concreted lot, and

$$W = 0.83\ R\ , \text{ and} \tag{26}$$

$$W = 1.38\ R - 0.34 \tag{27}$$

for the nonconcreted lot. The units of W and R are the same as those in Equations (22) and (23). Further investigation of these results shows that the organic material of the unconcreted lot is relatively unsoluble as compared to the concrete lot, due to the smaller values of K_1 and K_2 in the injection equations. This decreased solubility on the nonsurfaced lot can be attributed to the effect of the "binding properties" of the soil.

The values of R_c for the three cases are calculated to be 0.196 (in./hr.), 0.088 (in./hr.), and 0.246 (in./hr.) respectively. They are in reasonable agreement with the observed value (0.06 - 0.6 in./hr.) as reported by Miner (1). Furthermore, a larger value of R_c would be expected for the unsurfaced lot due to the

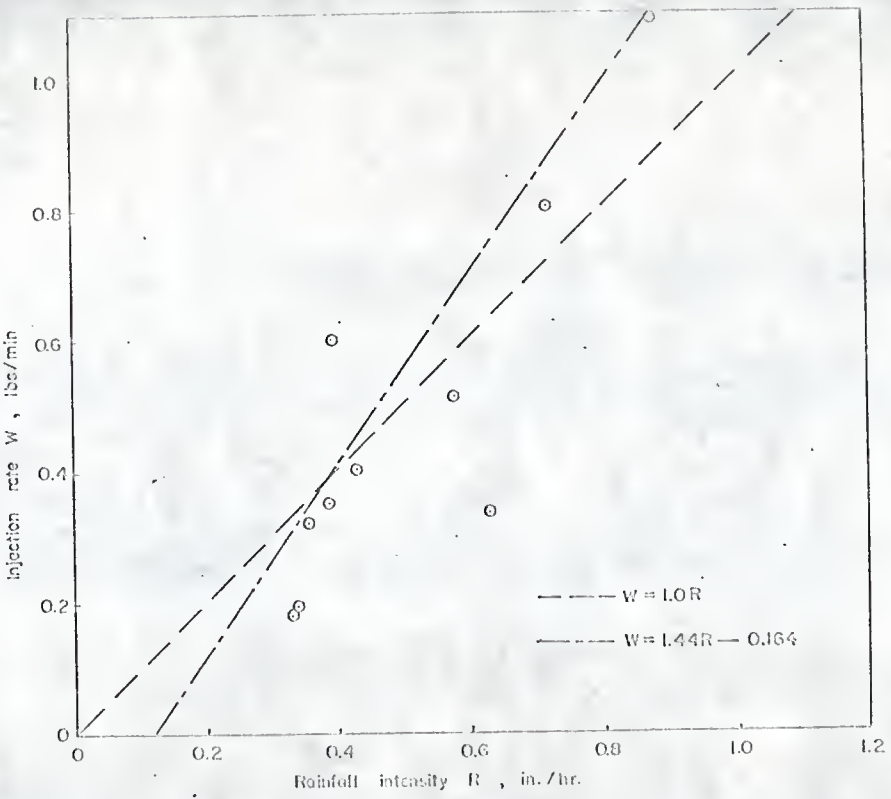


Fig. 10. Injection rate vs. rainfall intensity.

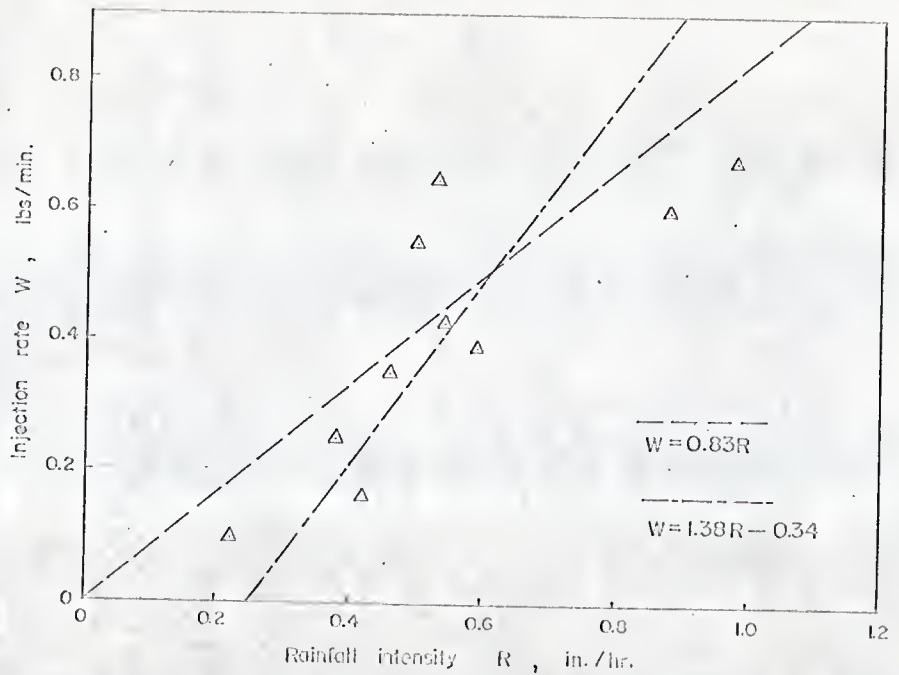


Fig. 11. Injection rate vs. rainfall intensity.

infiltration processes; the results do reveal this fact.

Considering the physical system which has been described, the diffusion mechanism should be taken into account. In the case, however, that the manure surface is not covered completely by water, or that the water layer on top of the lot surface is very thin, the mass transfer phenomenon in the feedlot system can be considered to be caused mainly by the turbulence of flow over manure. Accordingly, it is reasonable to presume that rainfall impact is the major factor. This suggested model may be called the "Rainfall Impact Model of Injection Rates".

The values of weir height δ obtained from data fitting range from 0.55 to 0.1. It may be expected that the value of δ should be some function of the surface condition of the feedlot. Unfortunately, due to the complexity of the system, no adequate scheme has been found to correlate the parameter to the physical system.

The values of initial concentration of COD are found to be in the range from 1.2 to 2.0, with most values lying around 0.45.

Further consideration of Equation (3) shows that the right-hand side dropped out at steady state conditions. Therefore, we obtain

$$W = C\,Q \tag{28}$$

for the steady state condition. If the pollution potential of a system is to be considered both qualitatively and quantitatively, the product of COD and outflow rate Q may be regarded as indicating the pollution strength more adequately. In this sense, the injection rate W which equals this product at the steady state

condition may be considered as the most important characteristic of the system. The curve of CQ is shown in Figure 12 as an illustration.

The preceding argument may not be too conclusive. This is because data of field studies are usually scattered, and fitting these data to the analog solution by eye has the potential of introducing further error. However, as compared to the basic variability property of the feedlot system, it is not considered a serious weakness.

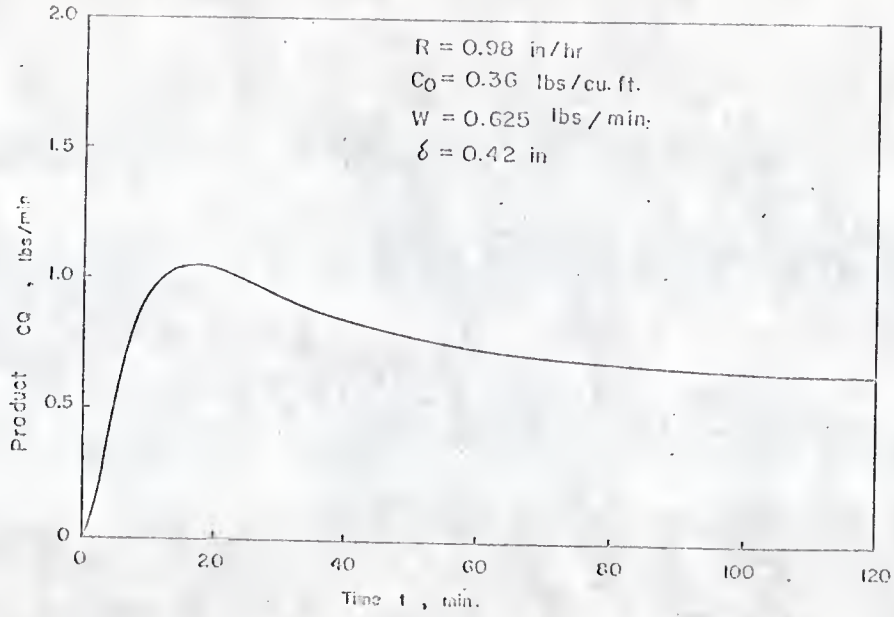


Fig. 12. Product of concentration and flow rate vs. time.

DIMENSIONAL ANALYSIS AND SIMULATION STUDY OF THE SYSTEM

Dimensional analysis is a technique of applying the principle of similarity to physical systems. The principle states that "the spatial and temporal configuration of a physical system is determined by ratio of magnitudes within the system itself and does not depend upon the size or nature of the units in which these magnitudes are measured" (14). In this section, the system equations are rewritten in dimensionless form to obtain the characteristic dimensionless groups. These dimensionless groups can then be used to obtain a better prediction of the system's behavior.

The fluctuation of rainfall intensity as a function of time may also be taken into account in simulation studies by using dimensionless systems equations. The response of the system to four different time input functions is considered. These four functions are:

(1) Sinusoidal function,

(2) Superposition of three sinusoidal functions,

(3) Square-wave function, and

(4) Function of random variable

Both digital and analog computers are employed in this study.

Dimensionless Form of The System Equations

Before converting the system equations into dimensionless form, a further assumption may be made. The injection equations

$$W = K_1 R \tag{29}$$

and $$W = K_2 R + K_3. \tag{30}$$

It is reasonable to postulate that the injection rate $\dot{W}$ is proportional to the size of the feedlot surface. Then the injection equations can be written in the forms

$$W = Ak_1 R \tag{31}$$

and

$$W = A\, k_2 R - Ak_3 \tag{32}$$

where A is the surface area of the feedlot. If the rainfall intensity in Equations (31) and (32) is expressed in terms of ft/min, we obtain

$$k_1 = \frac{K_1}{A} \times 720 \tag{33}$$

$$k_2 = \frac{K_2}{A} \times 720 \tag{34}$$

$$k_3 = \frac{-K_3}{A} \times 720. \tag{35}$$

where the values of K_1, K_2 and K_3 have been given in the preceding section. The foregoing assumption implies that the values of k_1, k_2 and k_3 do not change with feedlot surface area.

Substituting of Equations (31) and (32) into Equation (3), we obtain:

$$V \frac{dC}{dt} = \begin{cases} Ak_1 R - CQ & \text{type (I)} \quad (36) \\ Ak_2 R - Ak_3 - CQ & \text{type (II)} \quad (37) \end{cases}$$

Equation (36) or (37) together with Equations (1), (4), and (5) are then employed for the dimensional analysis of the system.

The following dimensionless variables may be defined

$$\theta = \frac{t}{T}\ ,\quad r = \frac{R}{R_s}\ ,\quad q = \frac{Q}{AR_s}\ ,\quad v = \frac{V}{TAR_s}$$
$$h = \frac{H}{R_s T}\quad c = \frac{C}{C^*} \tag{38}$$

where the lower case letters represent the dimensionless variables. We also define:

T = a characteristic time of the system

R_s = average rainfall intensity

C^* = a characteristic concentration.

To avoid repetition, only the injection rate equation of type (II), i.e., Equation (37) is used in the following derivation. Equations (1), (4), (5), and (37) can be rewritten in terms of the foregoing dimensionless variables, by setting $t = \theta T$, $r = rR_s$, $Q = qAR_s$, $V = vTAR_s$, $H = hR_sT$ and $C = cC^*$,

$$q = \left(\frac{\lambda L}{AR_s}\,(TR_s)^{3/2}\right) h^{3/2} \tag{39}$$

$$v = h + \left(\frac{\delta}{TR_s}\right) \tag{40}$$

$$\frac{dh}{d\theta} = r - q \tag{41}$$

$$v\frac{dc}{d\theta} = \left(\frac{k_2}{C^*}\right)r - \frac{k_3}{C^* R_s} - cq \tag{42}$$

If the characteristic concentration C^* in Equation (38) is chosen to be k_2, Equation (42) can be simplified to

$$v \frac{dc}{d\theta} = r - \frac{k_3}{k_2 R_s} - cq \qquad (43).$$

Furthermore, the characteristic time T may be defined by letting the steady state dimensionless tank volume $v_s = 1$. Thus from Equation (38), T can be expressed in an explicit form as

$$T = \frac{V_s}{AR_s} \qquad (44)$$

or

$$T = \frac{\left[\left(\frac{AR_s}{\lambda L}\right)^{2/3} + \delta\right]}{R_s} \qquad (45)$$

where V_s is the steady-state tank volume under constant rainfall intensity. Equation (45) is obtained by recognizing that at the steady state condition,

$$Q_s = AR_s = \lambda L H_s^{3/2} \qquad (46)$$

and

$$V_s = A(H_s + \delta). \qquad (47)$$

T can thus be considered as some kind of residence time of the system.

Examination of Equations (39), (40), (41) and (43), gives rise to the following dimensionless groups, namely,

$$\alpha = \left(\frac{k_3}{k_2 R_s}\right) \qquad (48)$$

$$\beta = \frac{\delta}{TR_s} = \frac{\delta}{\left[\left(\frac{AR_s}{\lambda L}\right)^{2/3} + \delta\right]} \qquad (49)$$

$$\gamma = \frac{\lambda L}{AR_s} (TR_s)^{2/3} \qquad (50)$$

or

$$\gamma = \frac{\lambda L}{AR_s}\left[\left(\frac{AR_s}{\lambda L}\right)^{3/2} + \delta\right]^{2/3} \tag{51}$$

A further algebraic manipulation shows that γ can be expressed in terms of β by the following equation

$$\gamma = \frac{1}{(1-\beta)^{3/2}} \tag{52}$$

Thus β and γ are dependent.

In these dimensionless equations the "scale factors", that is, A, L, R_s, and δ which appear in the original equations are concentrated in two dimensionless groups α and β. Two different systems with the same values of α, β and dimensionless initial conditions are said to be "dynamically similar"; that is, the dimensionless responses of the system are the same in each case.

If the injection equation of type (I), i.e., Equation (31) is considered, we have only one independent dimensionless group, α being equal to zero since $k_3 = 0$. This can be seen by comparing Equations (36) and (37). The system can thus be identified by one parameter β with a specified initial concentration. The characteristic concentration C* is chosen to be k_1 in this case.

The numerical ranges of α and β defined in Equations (48) and (49) can be determined from the following considerations:

(1) Since the case of $R_s < \frac{k_3}{k_2}$ must be excluded from a practical point of view as discussed in a previous section, and values of k_2, k_3, and R_s are all positive quantities, the range of α should, therefore, be between zero and one, that is

$$0 < \alpha < 1 \tag{53}$$

(2) Since δ and $\left(\frac{AR_s}{\lambda L}\right)^{2/3}$ in Equation (49) are both positive quantities, the range of β is obviously

$$0 < \beta < 1 \tag{54}$$

Analog Solution of Dimensionless Equations

The dimensionless system equations are summarized as follows:

$$q = \frac{1}{(1-\beta)^{3/2}} h^{3/2} \tag{55}$$

$$v = h + \beta \tag{56}$$

$$\frac{dh}{d\theta} = r - q \qquad h = 0 \quad \text{at } \theta = 0 \tag{57}$$

$$v \frac{dc}{d\theta} = \begin{cases} r - cq, & \text{type (I)} \\ r - \alpha - cq, & \text{type (II)} \end{cases} \tag{58, 59}$$

where $r = 1$ for constant rainfall intensity.

The dimensionless system equations have essentially the same form as the original system equations. They are nonlinear. The analog solutions can be obtained for various values of α and β and a specified initial concentration. The analog computer circuit is shown in Figure 13. The analog solution of dimensionless outflow rate q for $r = 1$ (constant rainfall) is shown in Figure 14 and the dimensionless concentration c for $r = 1$ (constant rainfall) are shown in Figures 15, 16, and 17. Initial concentrations of 1.0 and 1.6 are employed for injection equations of types (I) and (II) respectively. The following remarks can be made by referring to Figures 14, 15, 16, and 17.

(1) In Figure 14, the time of dimensionless flow rate q to

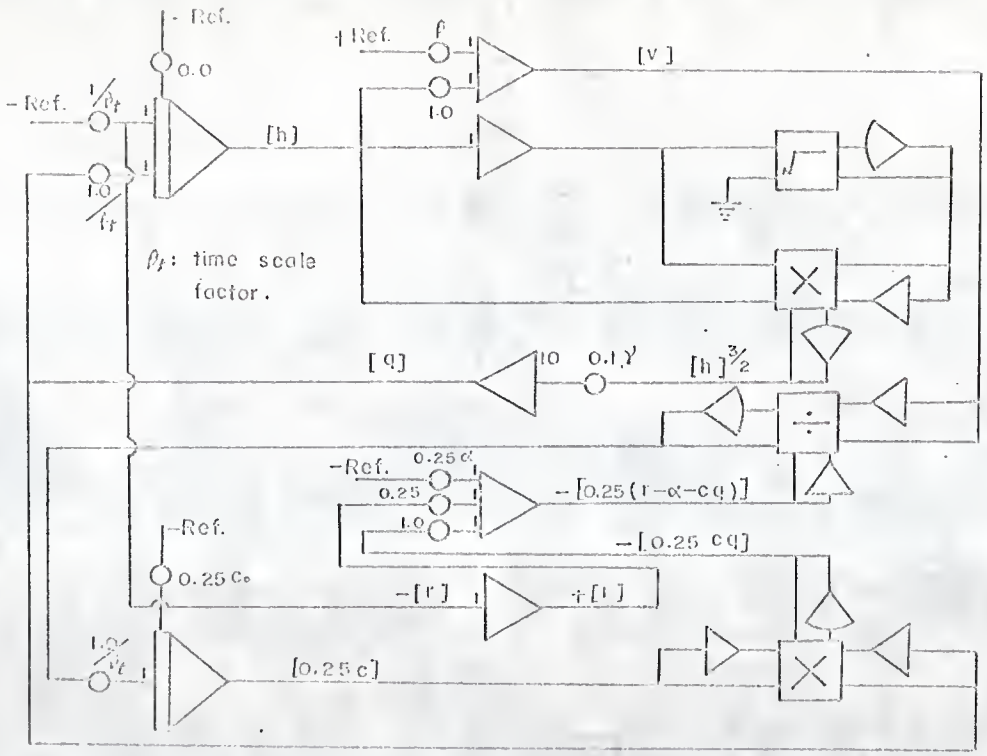


Fig.13. Analog computer circuit for solving dimensionless system equations.

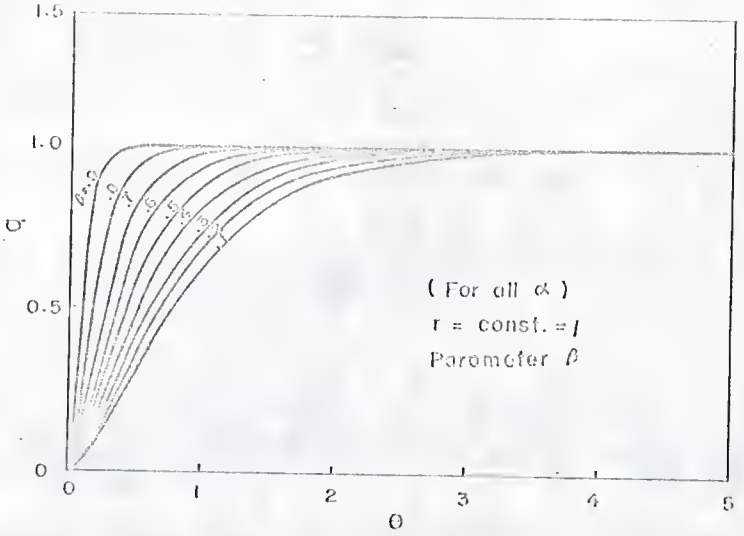


Fig.14. Dimensionless flow rate vs. dimensionless time for various values of β.

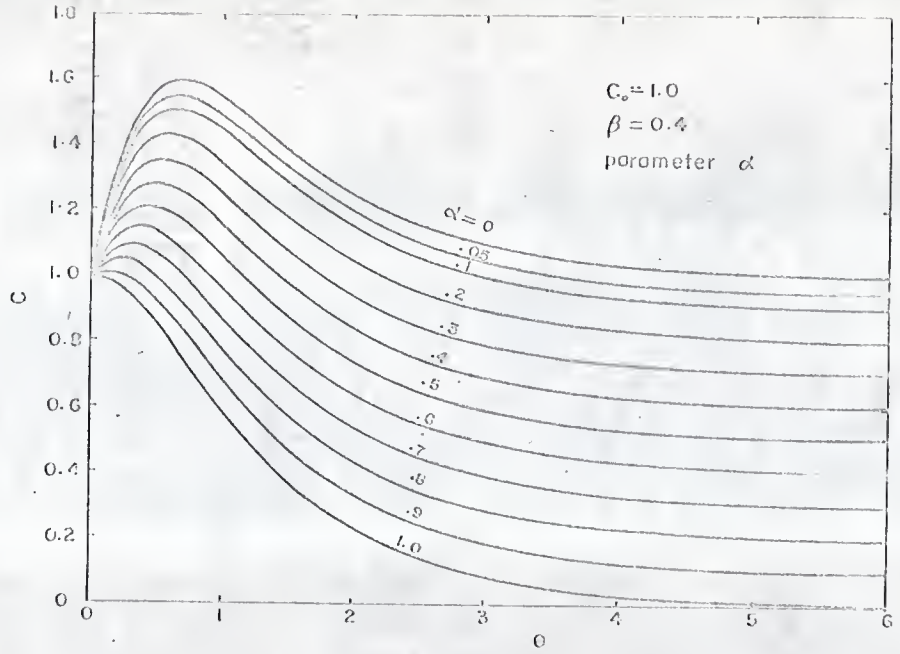


Fig. 15. Dimensionless concentration vs. dimensionless time for various values of α.

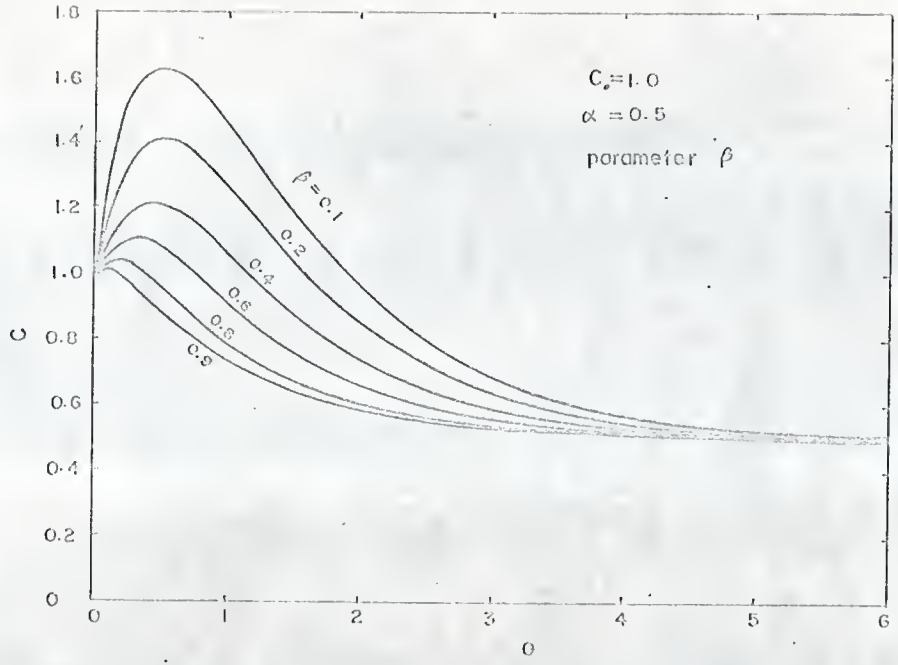


Fig. 16. Dimensionless concentration vs. dimensionless time for various values of β.

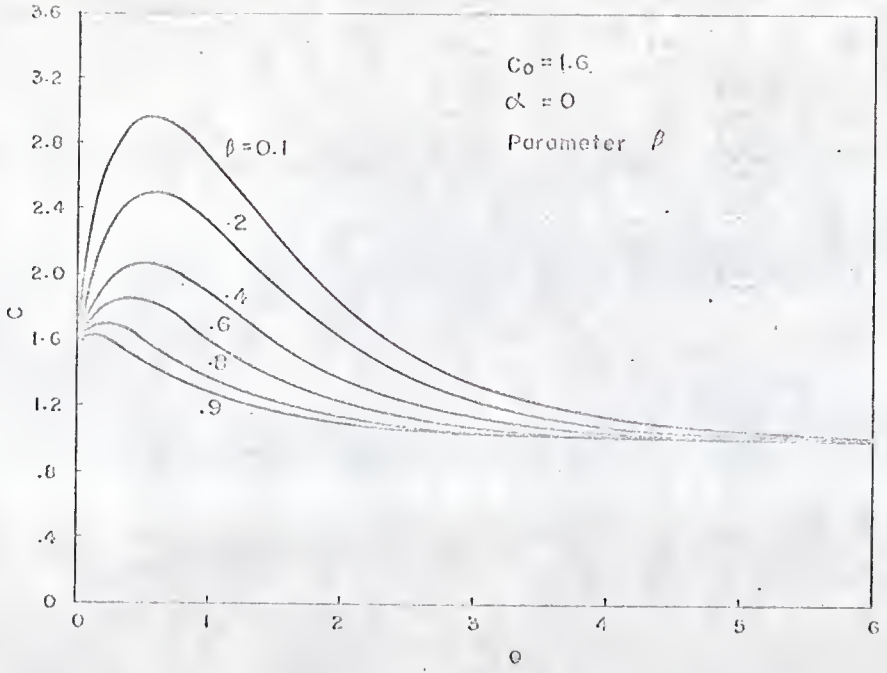


Fig. 17. Dimensionless concentration vs. dimensionless time for various values of β.

reach the steady state value increases as β is decreased. The steady state value is one for all cases. Furthermore, the flow rate q is independent of α as can be seen from dimensionless equations. The figure is identical for both types of injection rate equations.

(2) In Figure 15, the time to reach the peak concentration of COD increases as the value of α is decreased. However, the time to reach the steady state value is the same for all values of α. The steady state concentration for each case tends to the value of $(1 - \alpha)$. This can also be seen from Equation (59) by setting the left Hand side equal to zero and recalling that the steady state flow rate q equals one for all cases.

(3) In Figure 16, the peak concentration, the time of occurrence of peak concentration, and the time to reach the steady-state condition all increase as β is decreased. It is also seen that the steady-state concentration is independent of β.

(4) In Figure 17, the injection equation of type (I) is employed. The shape of the curve is very similar to those of type (II). The steady state concentrations for various values of β all tend to the value one, as can be expected from Equation (58). The peak concentration and the time to reach the peak concentration increase as β is decreased.

It may be concluded that smaller values of α and β indicate heavier pollution of water. This implies that higher rainfall intensity would cause heavier polluted runoff. Equation (51) indicates that for constant δ, an increase of $(\frac{AR}{\lambda L}^{S})$ would decrease the parameter β and thus consequently increase the concentration

at the initial stage. If the value of $(\frac{AR_s}{\lambda L})^{2/3}$ is of several orders of magnitude greater than δ, a decrease of δ would decrease β. On the other hand, if $(\frac{AR_s}{\lambda L})^{2/3}$ is of several orders of magnitude smaller than δ, a change of δ or $(\frac{AR_s}{\lambda L})^{2/3}$ would not affect β considerably.

System Response against Time-Varying Rainfall Intensities

Constant rainfall intensity during the runoff period is considered in the preceding sections. This is merely a simple approximation of the real system.

Justification of this assumption can be made by using a simple time-varying function of simulated rainfall as an input to the system. This varying function can be a function of regular shape such as sinusoidal function, squarewave function, or superposition of two or more sinusoidal functions. It can also be a function of random variables, such as a function of a uniformly distributed random variable or a normal distributed random variable. In this section, four types of time-varying input functions are considered .

(1) The first case considered is the sinusoidal input function. The dimensionless rainfall intensity is assumed to be a sine function with respect to time, which can be written as

$$r = r_s + b \sin\omega\theta \qquad (60)$$

where r_s = mean value or steady state component of rainfall intensity

b = amplitude of variation in rainfall intensity

ω = angular (radian) frequency, rad/time.

According to the preceding argument, it can be seen that the value of r_s is equal to one. Equation (60) has been substituted into Equations (57), (58), and (59) to obtain

$$\frac{dh}{d\theta} = 1 + b \sin\omega\theta - q \tag{61}$$

$$\frac{dc}{d\theta} = \begin{cases} 1 + b \sin\omega\theta - cq, & \text{type (I)} \\ 1 + b \sin\omega\theta - \alpha - cq, & \text{type (II)} \end{cases} \tag{62, 63}$$

Equations (55), (56), (61) combined with equation (62) or (63) can then be solved on an analog computer (Figure 18). The desired sine function is generated on an analog computer. The analog solutions of dimensionless flow rate q and concentration c together with rainfall intensity are shown in Figure 19 in which response of steady state rainfall is also presented for comparison. It shows that both flow rate and concentration fluctuate with the same frequency as the rainfall function. The amplitude of input fluctuation is greater than the output fluctuation. For example, in Figure 20, with $\omega = 10$ a 40% fluctuation of rainfall from the constant value results in a fluctuation of outflow rate of about 10% and a concentration fluctuation of less than 7% from the responses to constant rainfall input.

Different amplitude and frequency of rainfall functions have been used. As may be expected, increasing the amplitude and decreasing the radian frequency of the assumed rainfall function will increase the fluctuation of system response. This can be seen in Figure 20.

(2) The next input function considered is the superposition

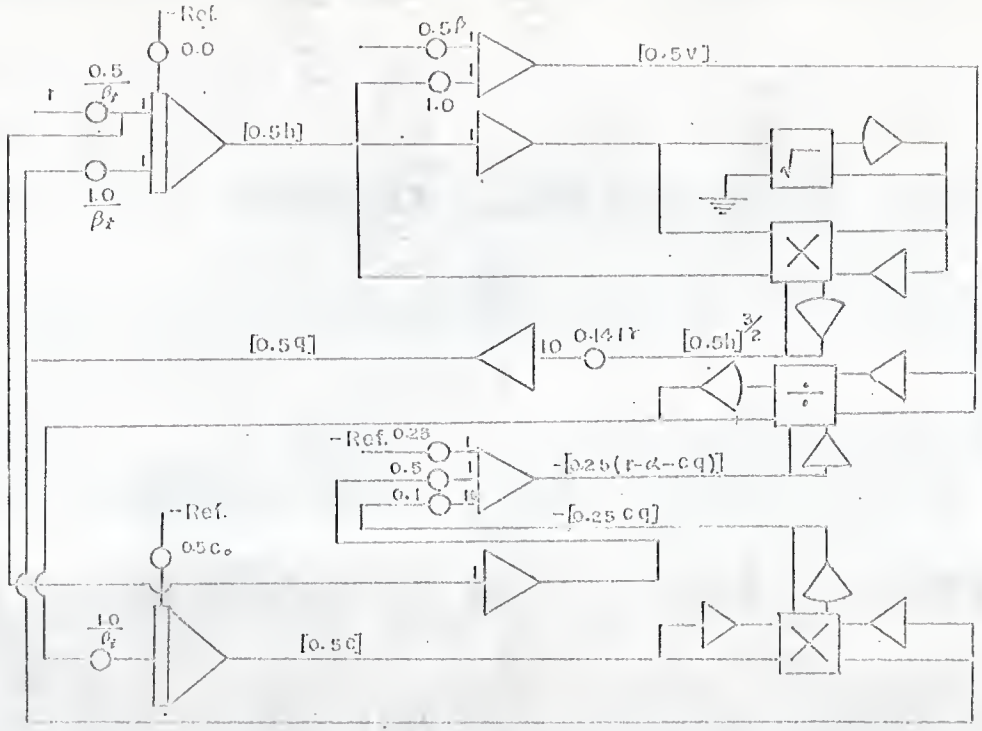


Fig. 18. Analog computer circuit for simulation study.

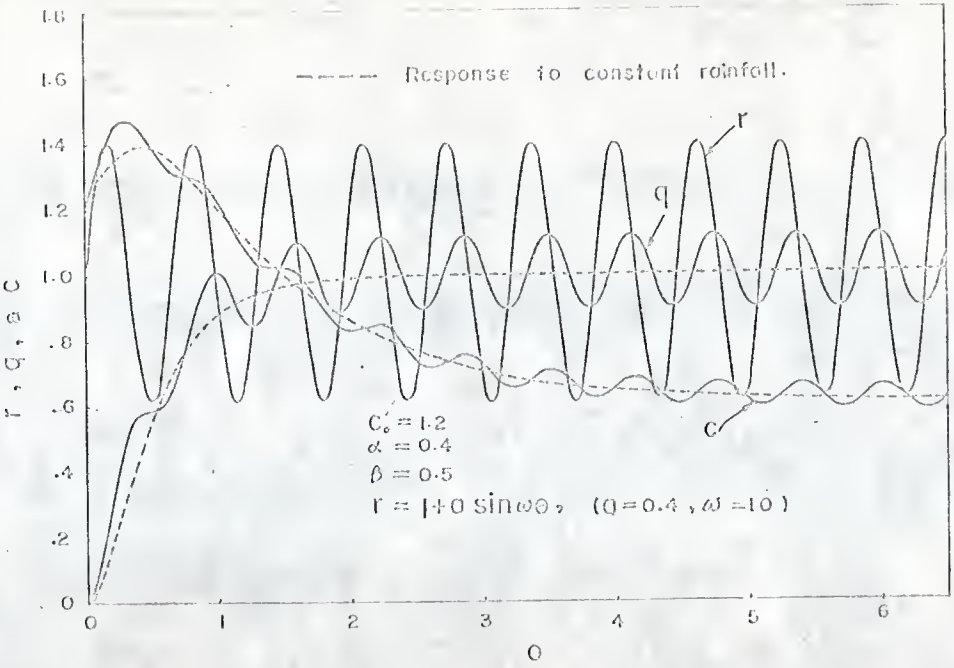


Fig. 19. - Response of the system to sinusoidal rainfall.

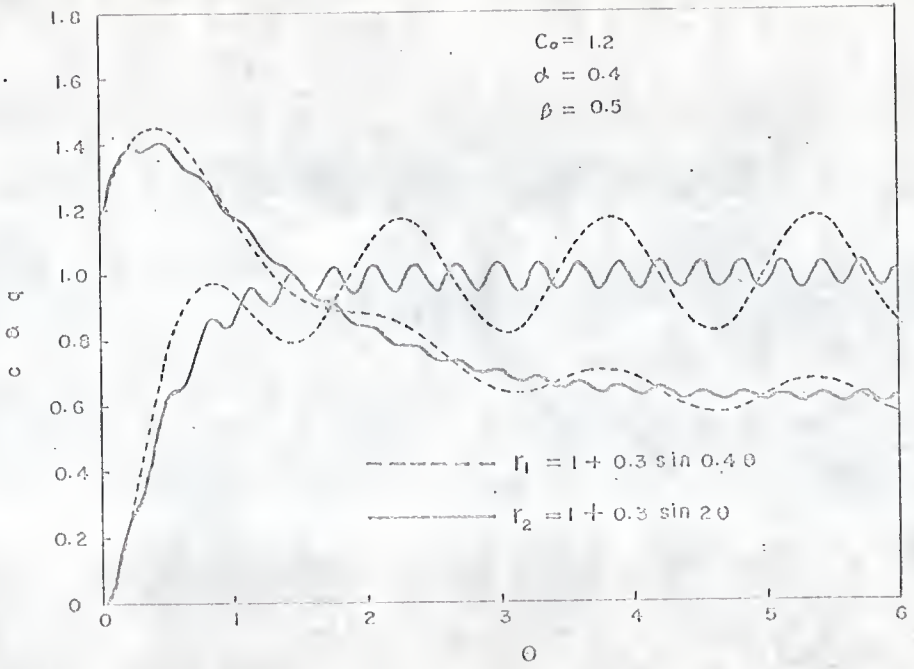


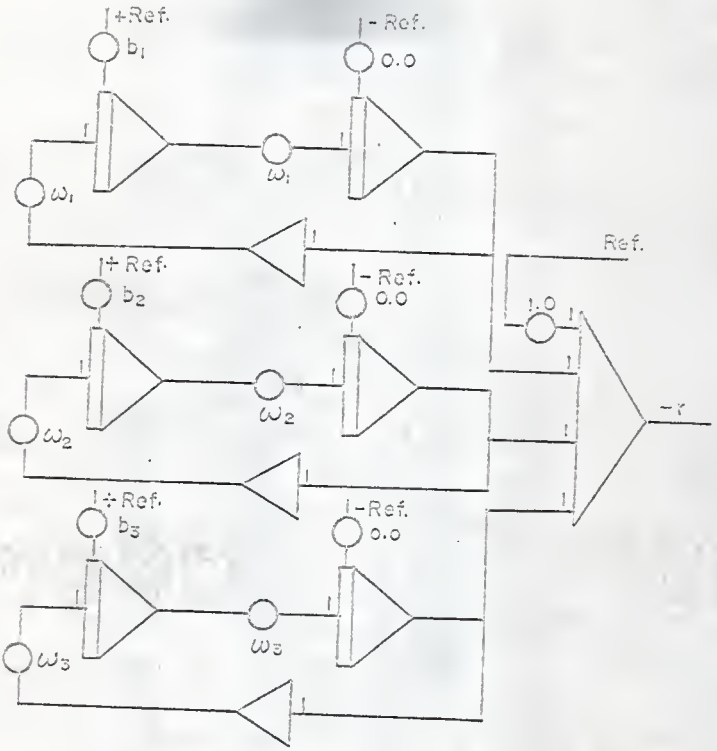
Fig. 20. Responses of the system to different frequency of sinusoidal rainfall.

of three sinusoidal functions. Three sine functions, each having different amplitude and frequency, are superimposed to produce a simulated random-like function; generation of this function is shown in Figure 21. The rainfall intensity and output response curves are shown in Figure 22. It is seen that the flow rate curve has a moderate deviation from that of the constant rainfall input case. The COD curve, however, deviates relatively slightly from the response of constant rainfall intensity. Figure 23 shows that the deviation of response curves decreases as the angular frequency of the rainfall function is increased.

(3) In the third case we consider the rainfall input as a square-wave function. In Figure 24, the response curve of square-wave functional rainfall is shown. Both flow rate and concentration curves are of zigzag shape. The square-wave functional input is generated by the functional switch on an analog computer.

(4) In the fourth case, rainfall intensity as a function of time is given by random variates. A particular outcome of an experiment, that is, a numerical or sample value of a random variable, is called a "random variate". Rainfall pattern is assumed to be a random process. Generation of pseudorandom variates was carried out on an IBM computer 360 by using the multiplicative congruential method. Random variates with different distributions (for example, normal random variates) can be obtained from a set of uniformly distributed random numbers. A brief description of these techniques is summarized in the Appendices. Further information can be found in Reference (11).

Substituting of Equations (55) and (56) into Equations (57)



$$r = 1 + b_1 \sin \omega_1 t + b_2 \sin \omega_2 t + b_3 \sin \omega_3 t .$$

Fig. 21. Generation of three superimposed sinusoidal functions using amplifiers.

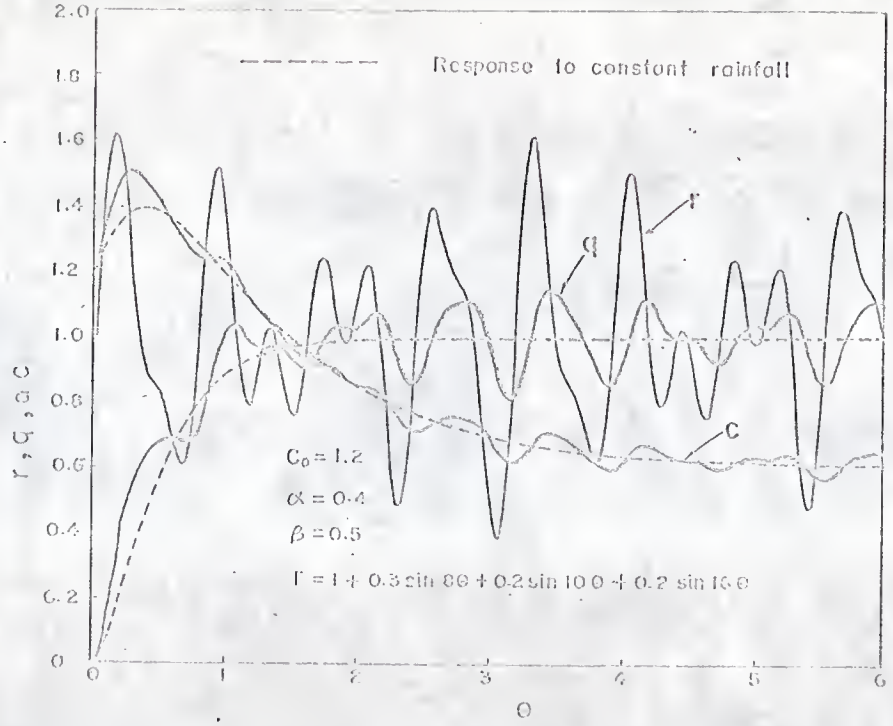


Fig. 22. Response of the system to superimposed sinusoidal functions.

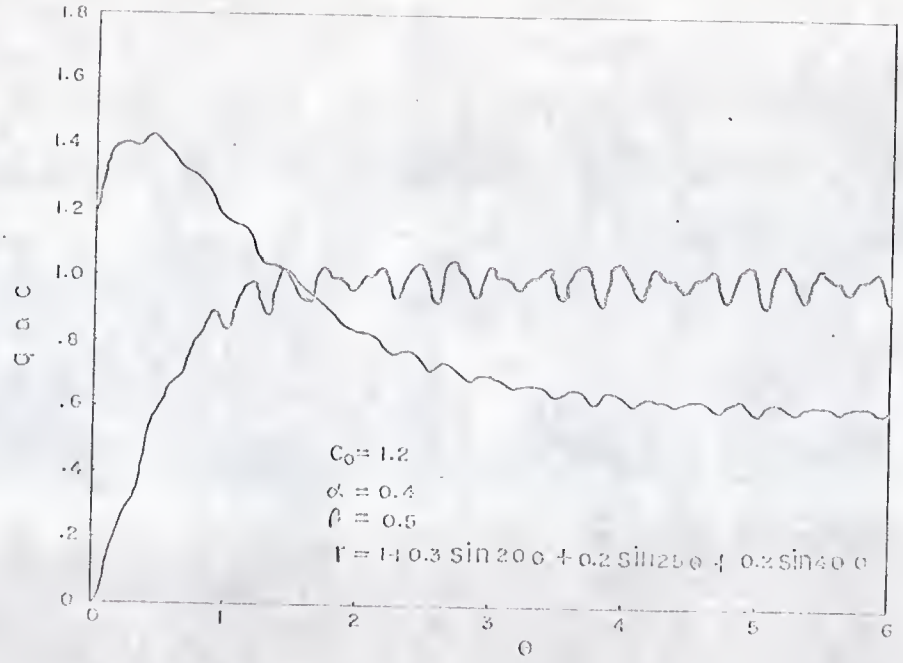


Fig. 23 : Response of the system to superimposed sinusoidal rainfall.

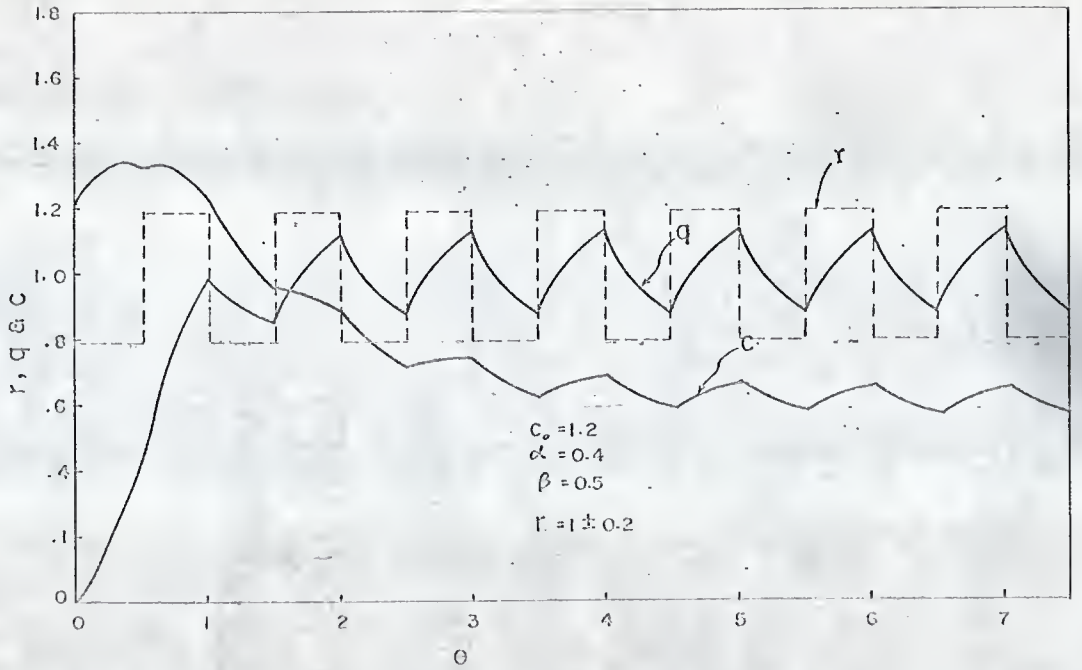


Fig. 2.4- Response of the system to square-wave rainfall pattern.

and (59), we obtain

$$\frac{dh}{d\theta} = r - \frac{1}{(1-\beta)^{3/2}} h^{3/2}, \qquad h = 0 \quad \text{at} \quad \theta = 0 \tag{64}$$

$$\frac{de}{d\theta} = \frac{1}{(1+\beta)} \left[r - \alpha - \frac{1}{(1-\beta)^{3/2}} eh^{3/2} \right], \quad c = c_o \text{ at } \theta = 0 \tag{65}$$

For a set of given values α, β, and e_o, Equations (64) and (65) are integrated simultaneously by employing the Runge-Kutta integration method with an increment in θ of 0.1. The random variates with desired distributions are generated and supplied to the main program where numerical integrations of Equations (64) and (65) are carried out. These random variates are supplied in two different ways in order to affect the frequency change of the oscillating function. In the first, one variate is supplied for each increment of θ; in the second, one variate is supplied for every two increments of θ of the numerical integration.

Random variates with uniform distribution and normal distribution are employed. Computed results of dimensionless concentration and outflow rate q are shown in Figures 25, 26, 27, and 28 for different cases. Out-flow rate q is calculated from Equation (55). It can be seen that the computed solutions depend on the variances of the random variates and the time interval in which random variates are supplied. Increasing the variance of random variates and this time interval will increase the deviation of the system responses from those of the constant input (rainfall).

The computer flow chart and computer program for this calculation using normal random variates are given in the Appendices.

Conclusion

In the first part, the method of dimensional analysis is used to analyse the system. The group of curves in Figures 14 through 17 may be used to predict the unknown flow rate and COD concentration in terms of two (or one) dimensionless characteristic parameters α and β (or β only) for various systems. These parameters are related through Equations (48) and (49) to the quantities A, R_s L, k_1, k_2, and weir height δ; some of them are measurable. The difficulty of the problem is the inability to correlate the weir height, δ. and the initial concentration to the physical system. These two variables affect the transient behavior of the system at the initial stage as can be seen in Figures 6 and 8.

In the simulation study, the system response to the time-varying rainfall intensity is observed. It is true in every case that increasing the amplitude of the deviation from the mean value and decreasing the frequency of change of the input function would increase the deviation of the system response from that of the constant input (rainfall). If the rainfall function of the real system is deviated moderately from the mean value and the frequency of change is in a proper range, the response of the system is essentially very close to that of the constant input. Therefore, a constant rainfall intensity is a good approximation. In the case of artificial rainfall events, especially, it can be treated as a steady state function with respect to time.

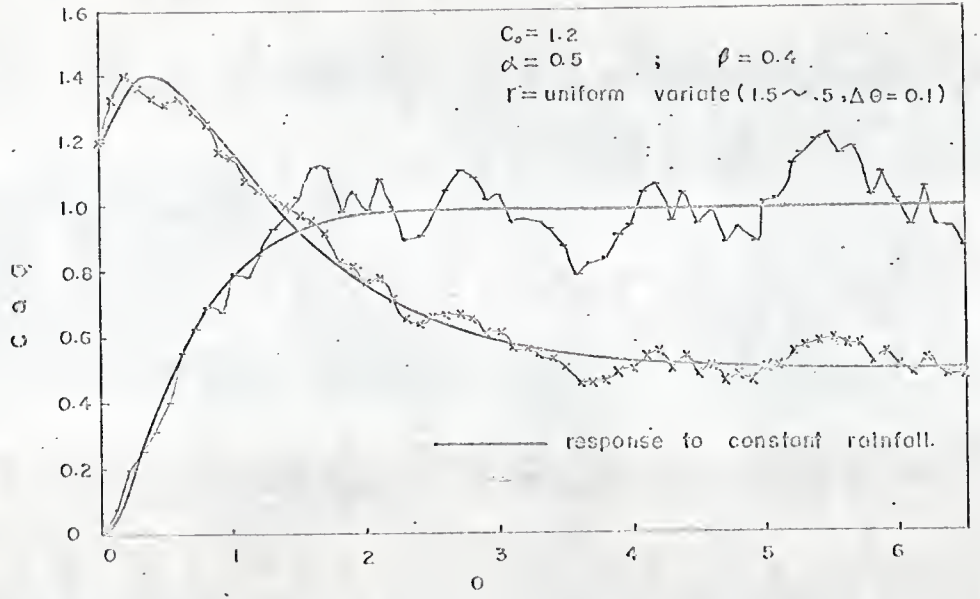


Fig. 25. Response of the system to random rainfall pattern (uniform distribution) with response to constant rainfall indicated.

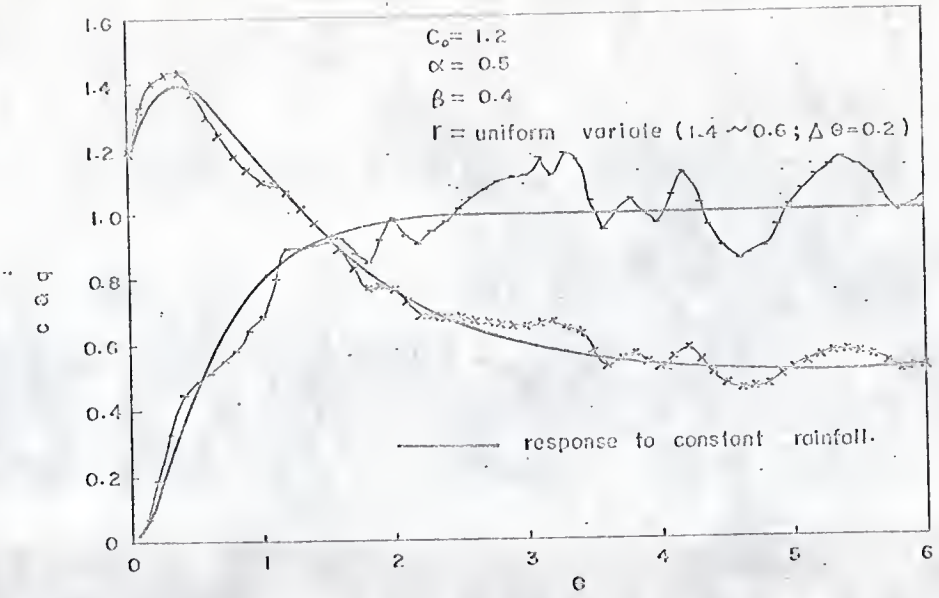


Fig. 26. Response of the system to random rainfall pattern (uniform distribution) with response to constant rainfall indicated.

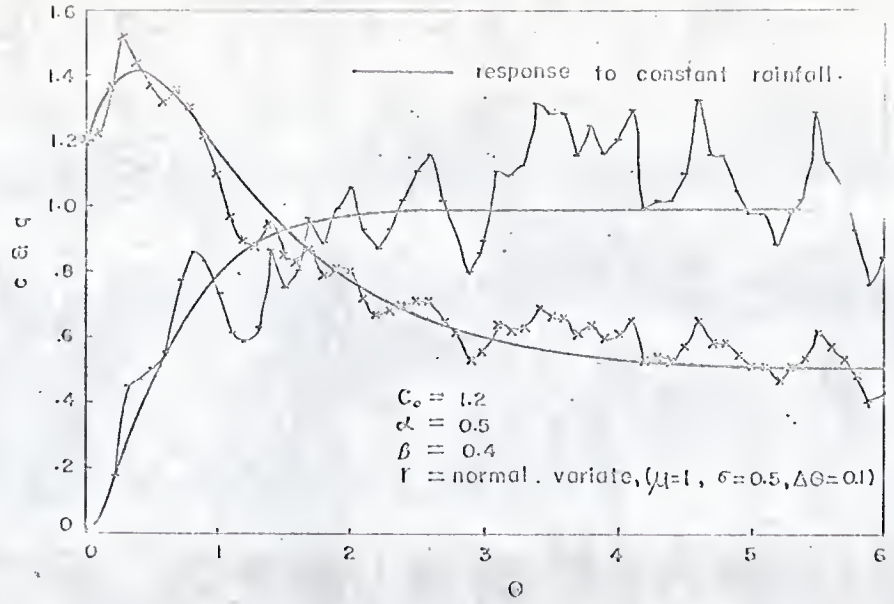


Fig. 27. Response of the system to random rainfall pattern (normal distribution) with response to constant rainfall indicated.

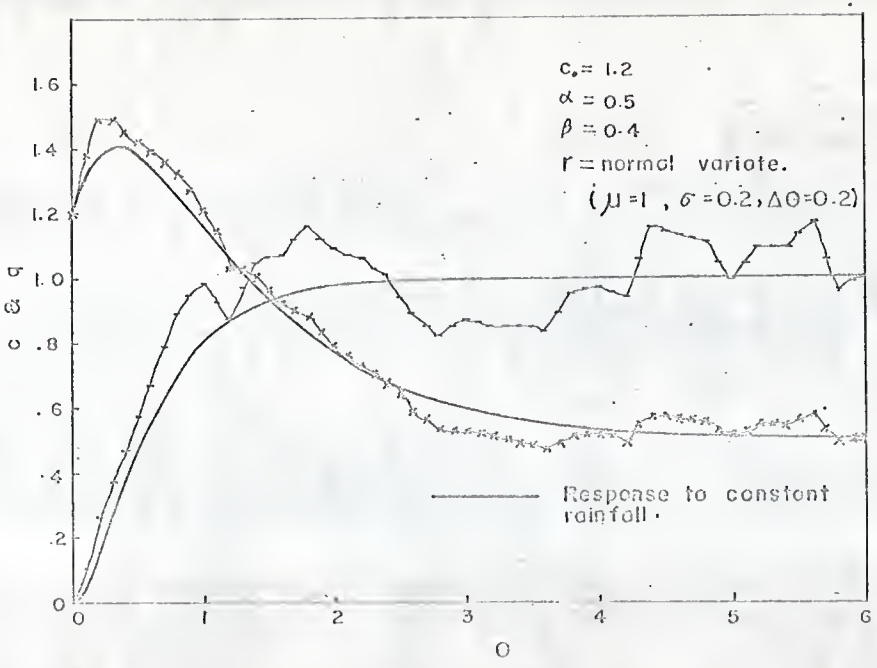


Fig. 28. Response of the system to random rainfall pattern (normal distribution) with response to constant rainfall indicated.

QUASILINEARIZATION TECHNIQUE FOR PARAMETERS RECOVERY

Quasilinearization is originated from Kantorovich's extension of the well-known Newton-Raphson procedure to function space which treats the non-linear problem as a limit of a sequence of linear problems (6). This technique has been well developed by Bellman and Kalaba (6) and has been widely used to solve non-linear boundary value problems in many fields of engineering. In this section, it is shown how the technique of quasilinearization can be applied, with some modifications, to the water pollution problem. The specific problem considered here is to obtain a set of "best fit" parameters of the feedlot system with the recovery of the initial condition for a given set of experimental data. Since in a complex system the observed initial data are not usually the true initial value of the system as described by the proposed model, this missing initial condition can be treated as an additional parameter of the system. The meaning of the term "best fit" depends on the particular criterion used. This choice, however, does not affect the development of the computational scheme in general.

Quasilinearization technique for parameter estimation was first used by Box and Hunter (29) in estimating kinetic coefficients.

The most recent papers concerning the curve fitting, parameters determination, and problem identification are those by Van Leeds (5), Donnelly and Quon (16) and Lee (13). This technique is briefly described in the following subsection without going into a detailed discussion of the theoretical background.

General Description of the Technique

To simplify the derivation of the technique, the system to be discussed is assumed as being described by an ordinary differential equation with one parameter, that is

$$\frac{dy}{dx} = g(f, a, y) \tag{66}$$

where f = a known forcing function; it may be a function of the independent variable x

a = parameter of the system

y = state or dependent variable of the system

Equation (66) is first linearized by a truncated Taylor's series expansion omitting the second and higher-order terms, i.e.

$$\frac{dy}{dx} = g(f, a^*, y^*) + \frac{\partial g}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}} (a - a^*) + \frac{\partial g}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}} (y - y^*) \tag{67}$$

$$\frac{dy}{dx} = \left[\frac{\partial g}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}}\right] a + \left[\frac{\partial g}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}}\right] y + \left[g(f, a^*, y^*) - \frac{\partial g}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}} a^* - \frac{\partial g}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}} y^*\right] \tag{68}$$

where a* and y* are the points about which the Taylor's series are expanded.

Note that Equation (68) is a first-order linear differential equation in terms of y and a. If parameter a is a constant which is independent of x, the following linear differential equation can be written

$$\frac{da}{dx} = 0 \tag{69}$$

Equation (69) merely shows that the derivative of the constant parameter is zero. Thus a pair of linear differential equations has to be solved. Because of the additive property of a linear system (the superposition principle) the general solution of these two equations can be written as the sum of a particular solution and two homogeneous solutions, that is

$$y(x) = y^p(x) + C_1 y_1^h(x) + C_2 y_2^h(x) \tag{70}$$

$$a(x) = a^p(x) + C_1 a_1^h(x) + C_2 a_2^h(x) \tag{71}$$

where the superscripts p and h represent the particular and homogeneous solutions of y and a respectively, and C_1 and C_2 are two arbitrary constants.

The particular solution is obtained by solving

$$\frac{dy}{dx} = \left[\frac{\partial f}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}}\right]a + \left[\frac{\partial f}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}}\right]y + \left[g(f, a^*, y^*) - a^* \frac{\partial f}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}} - y^* \frac{\partial f}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}}\right] \tag{72}$$

$$\frac{da}{dx} = 0 \tag{73}$$

and the homogeneous solutions are obtained by solving

$$\frac{dy}{dx} = \left[\frac{\partial f}{\partial a}\bigg|_{\substack{a=a^* \\ y=y^*}}\right]a + \left[\frac{\partial f}{\partial y}\bigg|_{\substack{a=a^* \\ y=y^*}}\right]y \tag{74}$$

$$\frac{da}{dx} = 0 \tag{75}$$

simultaneously. Furthermore, if the initial values in solving the particular and homogeneous solutions are chosen to be

$$\begin{cases} y^p(0) = 0 \\ a^p(0) = 0 \end{cases} \tag{76}$$

$$\begin{cases} y_1^h(0) = 1 \\ a_1^h(0) = 0 \end{cases} \tag{77}$$

and

$$\begin{cases} y_2^h(0) = 0 \\ a_2^h(0) = 1 \end{cases} \tag{78}$$

respectively, Equations (70) and (71) can be rewritten as

$$y(x) = y^p(x) + y(0)\, y_1^h(x) + a(0)\, y_2^h(x) \tag{79}$$

$$a(x) = a(0) \tag{80}$$

where y(0) and a(0) are the initial conditions of the complete solutions of y and a respectively. They are also the initial values of the dependent variable (system response) and the parameter of the model to be determined. To obtain the particular and homogeneous solutions by solving Equations (72) and (73), and Equations (74) and (75) respectively, the numerical integration method can be employed (for example, the Runge-Kutta method of numerical integration). The complete solution is then calculated by Equations (79) and (80). The two constants a(0) and y(0) are chosen so as to make the complete solution the "best fit" of a set

of observations. To achieve this, an error ϵ_i is defined as

$$\epsilon_i = y_e(x_i) - y(x_i) \tag{81}$$

where $y_e(x_i)$ is the observation at the point of the independent variable $x = x_i$. The mathematical problem is posed as one of finding a(0) and y(0) such that some function of ϵ_i is minimized. For illustrative purposes, the conventional "least squares" is used in the following derivation. According to this criterion, the minimizing function is defined as

$$\Phi = \sum_{i=1}^{m} \epsilon_i^2 = \sum_{i=1}^{m} [y_e(x_i) - y(x_i)]^2 \tag{82}$$

where m denotes the total observations available for one experimental run. It is noted that the observations are usually taken at discontinuous points. The functional Φ is thus defined in a discrete manner. Equation (79) is substituted into Equation (82) to obtain

$$\Phi = \sum_{i=1}^{m} [y_e(x_i) - y^p(x_i) - y_1^h(x_i)y(0) - y_2^h(x_i)a(0)]^2 \tag{83}$$

The minimization of functional Φ can be carried out through the use of various minimization methods such as search technique and by the simple differentiation. When the differentiation method is employed, functional Φ is partially differentiated with respect to y(0) and a(0) and the resulting derivatives are set equal to zero, i.e.

$$\frac{\partial \Phi}{\partial y(0)} = 0 \tag{84}$$

$$\frac{\partial \Phi}{\partial a(0)} = 0 \tag{85}$$

This gives rise to the following equations.

$$\sum_{i=1}^{m}\Big\{[y^p(x_i) - y_e(x_i)]y_1^h(x_i) + [y_1^h(x_i)]^2 y(0) + y_2^h(x_i)\, y_1^h(x_i)\, a(0)\Big\} = 0 \tag{86}$$

$$\sum_{i=1}^{m}\Big\{[y^p(x_i) - y_e(x_i)]y_2^h(x_i) + y_1^h(x_i)\, y_2^h(x_i)\, y(0) + [y_2^h(x_i)]^2\, a(0)\Big\} = 0 \tag{87}$$

Equations (86) and (87) can be rewritten in matrix notation as

$$\begin{bmatrix} \sum_{i=1}^{m}[y_1^h(x_i)]^2 & \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) \\ \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) & \sum_{i=1}^{m}[y_2^h(x_i)]^2 \end{bmatrix} \begin{bmatrix} y(0) \\ a(0) \end{bmatrix} = - \begin{bmatrix} \sum_{i=1}^{m}[y^p(x_i)-y_e(x_i)]y_1^h(x_i) \\ \sum_{i=1}^{m}[y^p(x_i)-y_e(x_i)]y_2^h(x_i) \end{bmatrix} \tag{88}$$

$y(0)$ and $a(0)$ are then solved by Cramer's rule, as follows

$$y(0) = - \frac{\begin{vmatrix} \sum_{i=1}^{m}[y^p(x_i) - y_e(x_i)]y_1^h(x_i) & \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) \\ \sum_{i=1}^{m}[y^p(x_i) - y_e(x_i)]y_2^h(x_i) & \sum_{i=1}^{m}[y_2^h(x_i)]^2 \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^{m}[y_1^h(x_i)]^2 & \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) \\ \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) & \sum_{i=1}^{m}[y_2^h(x_i)]^2 \end{vmatrix}} \tag{89}$$

$$a(0) = \frac{\begin{vmatrix} \sum_{i=1}^{m} [y_1^h(x_i)]^2 & \sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_1^h(x_i) \\ \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) & \sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_2^h(x_i) \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^{m} [y_1^h(x_i)]^2 & \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) \\ \sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i) & \sum_{i=1}^{m} [y_2^h(x_i)]^2 \end{vmatrix}} \tag{90}$$

or

$$y(0) = \frac{\left\{\sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_1^h(x_i)\right\} \sum_{i=1}^{m} [y_2^h(x_i)]^2}{\left\{\sum_{i=1}^{m} [y_1^h(x_i)]^2\right\}\left\{\sum_{i=1}^{m} [y_2^h(x_i)]^2\right\} - \left\{\sum_{i=1}^{m} y_1^h(x_i)\, y_2^h(x_i)\right\}^2}$$

$$- \frac{\left\{\sum_{i=1}^{m} y_1^h(x_i) y_2^h(x_i)\right\}\left\{\sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_2^h(x_i)\right\}}{\left\{\sum_{i=1}^{m} [y_1^h(x_i)]^2\right\}\left\{\sum_{i=1}^{m} [y_2^h(x_i)]^2\right\} - \left\{\sum_{i=1}^{m} y_1^h(x_i) y_2^h(x_i)\right\}^2} \tag{91}$$

$$a(0) = \frac{\left\{\sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_2^h(x_i)\right\} \sum_{i=1}^{m} [y_1^h(x_i)]^2}{\left\{\sum_{i=1}^{m} [y_1^h(x_i)]^2\right\}\left\{\sum_{i=1}^{m} [y_2^h(x_i)]^2\right\} - \left\{\sum_{i=1}^{m} y_1^h(x_i) y_2^h(x_i)\right\}^2}$$

$$- \frac{\left\{\sum_{i=1}^{m} y_1^h(x_i) y_2^h(x_i)\right\}\left\{\sum_{i=1}^{m} [y^p(x_i) - y_e(x_i)]\, y_1^h(x_i)\right\}}{\left\{\sum_{i=1}^{m} [y_1^h(x_i)]^2\right\}\left\{\sum_{i=1}^{m} [y_2^h(x_i)]^2\right\} - \left\{\sum_{i=1}^{m} y_1^h(x_i) y_2^h(x_i)\right\}^2} \tag{92}$$

where $y^p(x_i)$, $y_1^h(x_i)$, and $y_2^h(x_i)$ are calculated numerically. The parameter a(0) and the initial value y(0) are thus determined by Equations (91) and (92).

In Equation (68), the original system equation are linearized by omitting the second and higher-order terms of the Tayler's

series expansion. Therefore, the correct result for y(0) and a(0) can not be obtained in one step. The final solution of the desired accuracy is obtained by an iterative process.

For this purpose, the Taylor's series expansion in Equation (67) is rewritten in terms of the following recurrence relation,

$$\frac{dy_{n+1}}{dx} = g(f, a_n, y_n) + \frac{\partial g}{\partial a}\bigg|_{\substack{a=a_n \\ y=y_n}} (a_{n+1} - a_n) + \frac{\partial g}{\partial y}\bigg|_{\substack{a=a_n \\ y=y_n}} (y_{n+1} - y_n) \tag{93}$$

or

$$\frac{dy_{n+1}}{dx} = \frac{\partial g}{\partial a}\bigg|_{\substack{a=a_n \\ y=y_n}} a_{n+1} + \frac{\partial g}{\partial y}\bigg|_{\substack{a=a_n \\ y=y_n}} y_{n+1} + g(f, a_n, y_n)$$

$$- \frac{\partial g}{\partial a}\bigg|_{\substack{a=a_n \\ y=y_n}} a_n - \frac{\partial g}{\partial y}\bigg|_{\substack{a=a_n \\ y=y_n}} y_n \tag{94}$$

with $$\frac{da_{n+1}}{dx} = 0 \tag{95}$$

It is noted that y* and a* in Equation (67) are replaced by y_n and a_n respectively. The subscripts in Equations (93), (94) and (95) denote the number of iterations. The results y_n and a_n obtained in preceding calculations are then used to obtain the next improved solutions. To start the iterative procedure, a set of initial solutions of y_o and a_o must be estimated.

Summary of The Procedure

The procedure used in the quasilinearization techniques to determine the parameters and the missing initial condition of non-linear system, given a set of observations, can be summarized as

follows:

1. The set of equations which describe the system is first linearized by the Taylor's series expansion as shown in Equation (67).

2. The recurrence relation for the linearized differential equation is constructed using Equations (94) and (95).

3. The appropriate approximate function $y(x_1)$ and a are assumed as the initial trial solution.

4. Forming the appropriate nonhomogeneous and homogeneous equations, as shown in Equations (72) and (73) and Equations (74) and (75), these equations are then solved numerically for a set of given initial values.

5. The obtained homogeneous and particular solutions with given experimental data are then used to calculate y(0) and a(0) by Equations (91) and (92). The improved solutions are obtained by Equations (79) and (80).

6. The improved approximate solutions replace the initial trial solutions. Steps 4. and 5. are repeated until the solution converges to the desired accuracy.

Application of Quasilinearization Technique to the Feedlot System

The equations describing the feedlot system are rewritten below for convenience, they are

$$Q = \lambda L H^{3/2} \tag{96}$$

$$V = A(H + \delta) \tag{97}$$

$$\frac{dH}{dt} = R - \frac{Q}{A}, \qquad H = 0 \quad @\ t = 0 \tag{98}$$

$$V \frac{dC}{dt} = W - CQ, \qquad C = C_o \quad @\ t = 0 \tag{99}$$

Substituting Equations (96) and (97) in Equations (98) and (99), we obtain

$$\frac{dH}{dt} = R - 1.69\, H^{3/2} \qquad H = 0 \quad @\ t = 0 \tag{100}$$

$$\frac{dC}{dt} = .000452\, \frac{W}{(H+\delta)} - 1.69 \frac{CH^{3/2}}{(H+\delta)} \qquad C = C_o \quad @\ t = 0 \tag{101}$$

where the following numerical values are employed.

$A = 2210\ ft^2$

$L = 24\ ft$

$\lambda = 2.6 \times 60\ ft^{\frac{1}{2}}/min$

Equations (100) and (101) are linearized by the Taylor's series expansion truncated after the first order term and the following recurrence differential equations are constructed.

$$\frac{dH_{n+1}}{dt} = (R - 1.69H_n^{3/2}) - (H_{n+1} - H_n)(2.535)H_n^{\frac{1}{2}},$$

$$H_{n+1} = 0 \quad @\ t = 0 \tag{102}$$

$$\frac{dC_{n+1}}{dt} = \frac{1}{(H_n+\delta_n)}\Big\{\Big[.000452W_n - 1.69C_nH_n^{3/2}\Big] + .000452(W_{n+1} - W_n)$$

$$- 1.69(C_{n+1} - C_n)H_n^{3/2} + (H_{n+1} - H_n)\Big[-2.54C_nH_n^{\frac{1}{2}} - \Big(\frac{.000452W_n - 1.69C_nH_n^{3/2}}{(H_n+\delta_n)}\Big)\Big]$$

$$- (\delta_{n+1} - \delta_n)\Big[\frac{.000452W_n - 1.69C_nH_n^{3/2}}{(H_n + \delta_n)}\Big]\Big\} \tag{103}$$

Two additional equations are added. They are

$$\frac{dW_{n+1}}{dt} = 0 \tag{104}$$

$$\frac{d\delta_{n+1}}{dt} = 0 \tag{105}$$

Equations (104) and (105) show that the derivatives of the constant parameters are zero.

Equations (102), (103), (104), and (105) are four first order simultaneous differential equations with one known initial condition, $H_{n+1} = 0$ @ $t = 0$. By making use of the known initial condition, solutions of these four equations can be represented as the sum of a particular solution and three homogeneous solutions, namely,

$$H_{n+1}(t) = H^{p}_{n+1}(t) + \sum_{j=1}^{3} a_{j,n+1}\, H^{h}_{j,n+1}(t) \tag{106}$$

$$C_{n+1}(t) = C^{p}_{n+1}(t) + \sum_{j=1}^{3} a_{j,n+1}\, C^{h}_{j,n+1}(t) \tag{107}$$

$$W_{n+1}(t) = W^{p}_{n+1}(t) + \sum_{j=1}^{3} a_{j,n+1}\, W^{h}_{j,n+1}(t) \tag{108}$$

$$\delta_{n+1}(t) = \delta^{p}_{n+1}(t) + \sum_{j=1}^{3} a_{j,n+1}\, \delta^{h}_{j,n+1}(t) \tag{109}$$

where the superscripts p and h represent particular and homogeneous solutions respectively, and $a_{j,n+1}$'s are integration constants. It should be noted that both the particular and the homogeneous solutions can be obtained numerically using the Runge-Kutta method.

The particular solutions are obtained by solving the following equations,

$$\frac{dH_{n+1}}{dt} = -2.535\, H_n^{\frac{1}{2}} H_{n+1} + (R + 0.845\, H_n^{3/2}) \tag{110}$$

$$\frac{dC_{n+1}}{dt} = \frac{1}{(H_n+\delta_n)}\left\{\left[.000452W - 1.69C_n H_n^{3/2}\right] + .000452(W_{n+1} - W_n)\right.$$

$$- 1.69(C_{n+1} - C_n)H_n^{3/2} + (H_{n+1} - H_n)\left[-2.54\, C_n H_n^{\frac{1}{2}} - \frac{(.000452W_n - 1.69C_nH_n^{3/2})}{(H_n+\delta_n)}\right] - (\delta_{n+1} - \delta_n)\left[\frac{.000452W_n - 1.69C_nH_n^{3/2}}{H_n+\delta_n}\right]\Big\} \quad (111)$$

$$\frac{dW_{n+1}}{dt} = 0 \quad (112)$$

$$\frac{d\delta_{n+1}}{dt} = 0 \quad (113)$$

And the homogeneous solutions are obtained by solving

$$\frac{dH_{n+1}}{dt} = -2.535H_n^{\frac{1}{2}}\, H_{n+1} \quad (114)$$

$$\frac{dC_{n+1}}{dt} = \frac{1}{(H_n+\delta_n)}\Big\{.000452W_{n+1} - 1.69\, C_{n+1}\, H_n^{3/2} + H_{n+1}\left[-2.54\, C_nH_n^{\frac{1}{2}} - \frac{(.000452\, W_n - 1.69\, C_nH_n^{3/2})}{(H_n+\delta_n)}\right] - \delta_{n+1}\left[\frac{.000452\, W_n - 1.69\, C_nH_n^{3/2}}{H_n+\delta_n}\right]\Big\} \quad (115)$$

$$\frac{dW_{n+1}}{dt} = 0 \quad (116)$$

$$\frac{d\delta_{n+1}}{dt} = 0 \quad (117)$$

The set of initial values used for numerical integration to obtain the particular solution is chosen as

$$\begin{cases} H^{p}_{n+1}(0) = 0 \\ C^{p}_{n+1}(0) = 0 \\ W^{p}_{n+1}(0) = 0 \\ \delta^{p}_{n+1}(0) = 0 \end{cases} \tag{118}$$

And the three sets of initial values for solving homogeneous solutions are chosen as

(i)
$$\begin{cases} H^{h}_{1,n+1}(0) = 0 \\ C^{h}_{1,n+1}(0) = 1.0 \\ W^{h}_{1,n+1}(0) = 0 \\ \delta^{h}_{1,n+1}(0) = 0 \end{cases} \tag{119}$$

(ii)
$$\begin{cases} H^{h}_{2,n+1}(0) = 0 \\ C^{h}_{2,n+1}(0) = 0 \\ W^{h}_{2,n+1}(0) = 1.0 \\ \delta^{h}_{2,n+1}(0) = 0 \end{cases} \tag{120}$$

(iii)
$$\begin{cases} H^{h}_{3,n+1}(0) = 0 \\ C^{h}_{3,n+1}(0) = 0 \\ W^{h}_{3,n+1}(0) = 0 \\ \delta^{h}_{3,n+1}(0) = 1.0 \end{cases} \tag{121}$$

Choosing zero as the initial values of H in obtaining the particular and homogeneous solution satisfies the known initial condition of $H_{n+1}(0) = 0$. Furthermore, by letting $t = 0$ in Equations (107), (108), and (109), and using the initial values given in Equations (118), (119), (120), and (121), the initial concentration can be expressed as

$$C_{n+1}(0) = C^{p}_{n+1}(0) + \sum_{j=1}^{3} a_{j,n+1} C^{h}_{j,n+1}(0) \tag{122}$$

or

$$C_{n+1}(0) = a_{1,n+1} \tag{123}$$

and the two parameters W and δ can be expressed as

$$W_{n+1}(0) = W^{p}_{n+1}(0) + \sum_{j=1}^{3} a_{j,n+1} W^{h}_{j,n+1}(0) \tag{124}$$

or

$$W_{n+1}(0) = a_{2,n+1} \tag{125}$$

and

$$\delta_{n+1}(0) = \delta^{p}_{n+1}(0) + \sum_{j=1}^{3} a_{j,n+1} \delta^{h}_{j,n+1}(0) \tag{126}$$

or

$$\delta_{n+1}(0) = a_{3,n+1} \tag{127}$$

respectively. Therefore Equations (107), (108), and (109) can be rewritten as

$$C_{n+1}(t) = C^{p}_{n+1}(t) + C_{n+1}(0) C^{h}_{1,n+1}(t) + W_{n+1}(0) C^{h}_{2,n+1}(t) + \delta_{n+1}(0)\, C^{h}_{3,n+1}(t) \tag{128}$$

$$W_{n+1}(t) = W_{n+1}(0) \tag{129}$$

$$\delta_{n+1}(t) = \delta_{n+1}(0) \tag{130}$$

The missing initial concentration and two parameters are to be determined by fitting the numerical solution to a set of experimental data based on a certain criterion. The conventional "least squares" is used. According to this criterion, the minimizing function is defined as

$$\Phi_{n+1} = \sum_{s=1}^{m} [c^{p}_{n+1}(t_s) + C_{n+1}(0)c^{h}_{1,n+1}(t_s) + W_{n+1}(0)c^{h}_{2,n+1}(t_s) + \delta_{n+1}(0)c^{h}_{3,n+1}(t_s) - C_e(t_s)]^2 \tag{131}$$

where $C_e(t_s)$ denotes the experimental data observed at the time $t = t_s$, and "m" stands for the number of observations available. The constants $C_{n+1}(0)$, $W_{n+1}(0)$ and $\delta_{n+1}(0)$ are chosen so as to minimize the functional Φ_{n+1}. By letting

$$\frac{\partial \Phi_{n+1}}{\partial C_{n+1}(0)} = 0 \tag{132}$$

$$\frac{\partial \Phi_{n+1}}{\partial W_{n+1}(0)} = 0 \tag{133}$$

$$\frac{\partial \Phi_{n+1}}{\partial \delta_{n+1}(0)} = 0 \tag{134}$$

we obtain

$$\sum_{s=1}^{m} [c^{p}_{n+1}(t_s) + C_{n+1}(0)c^{h}_{1,n+1}(t_s) + W_{n+1}(0)c^{h}_{2,n+1}(t_s) + \delta_{n+1}(0)c^{h}_{3,n+1}(t_s) - C_e(t_s)]\, c^{h}_{1,n+1}(t_s) = 0 \tag{135}$$

$$\sum_{s=1}^{m}[c^{p}_{n+1}(t_s) + c_{n+1}(0)c^{h}_{1,n+1}(t_s) + W_{n+1}(0)c^{h}_{2,n+1}(t_s)$$

$$+ \delta_{n+1}(0)c^{h}_{3,n+1}(t_s) - c_e(t_s)]\, c^{h}_{2,n+1}(t_s) = 0 \qquad (136)$$

$$\sum_{s=1}^{m}[c^{p}_{n+1}(t_s) + c_{n+1}(0)c^{h}_{1,n+1}(t_s) + W_{n+1}(0)c^{h}_{2,n+1}(t_s)$$

$$+ \delta_{n+1}(0)c^{h}_{3,n+1}(t_s) - c_e(t_s)]\, c^{h}_{3,n+1}(t_s) = 0 \qquad (137)$$

After some simplification, Equations (135), (136) and (137) are rewritten as

$$c_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{1,n+1}(t_s)]^2 + W_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{2,n+1}(t_s)c^{h}_{1,n+1}(t_s)]$$

$$+ \delta_{n+1}(0)\sum_{s=1}^{m}c^{h}_{3,n+1}(t_s)c^{h}_{1,n+1}(t_s) + \sum_{s=1}^{m}[c^{p}_{n+1}(t_s)-c_e(t_s)]c^{h}_{1,n+1}(t_s) = 0 \qquad (138)$$

$$c_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{1,n+1}(t_s)c^{h}_{2,n+1}(t_s)] + W_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{2,n+1}(t_s)]^2$$

$$+\delta_{n+1}(0)\sum_{s=1}^{m}c^{h}_{3,n+1}(t_s)c^{h}_{2,n+1}(t_s)+\sum_{s=1}^{m}[c^{p}_{n+1}(t_s)-c_e(t_s)]c^{h}_{2,n+1}(t_s) = 0 \qquad (139)$$

$$c_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{1,n+1}(t_s)c^{h}_{3,n+1}(t_s)]+W_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{2,n+1}(t_s)c^{h}_{3,n+1}(t_s)]$$

$$+\delta_{n+1}(0)\sum_{s=1}^{m}[c^{h}_{3,n+1}(t_s)]^2+\sum_{s=1}^{m}[c^{p}_{n+1}(t_s)-c_e(t_s)]c^{h}_{3,n+1}(t_s) = 0 \qquad (140)$$

Applying Cramer's rule, we obtain

$$C_{n+1}(0) = \frac{\begin{vmatrix} A_4 & A_2 & A_3 \\ B_4 & B_2 & B_3 \\ D_4 & D_2 & D_3 \end{vmatrix}}{\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{vmatrix}} \tag{141}$$

$$W_{n+1}(0) = \frac{\begin{vmatrix} A_1 & A_4 & A_3 \\ B_1 & B_4 & B_3 \\ D_1 & D_4 & D_3 \end{vmatrix}}{-\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{vmatrix}} \tag{142}$$

$$\delta_{n+1}(0) = \frac{\begin{vmatrix} A_1 & A_2 & A_4 \\ B_1 & B_2 & B_4 \\ D_1 & D_2 & D_4 \end{vmatrix}}{\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{vmatrix}} \tag{143}$$

where $$A_1 = \sum_{s=1}^{m} [c^h_{1,n+1}(t_s)]^2 \tag{144}$$

$$A_2 = \sum_{s=1}^{m} [c^h_{2,n+1}(t_s)\ c^h_{1,n+1}(t_s)] \tag{145}$$

$$A_3 = \sum_{s=1}^{m} [c^h_{3,n+1}(t_s)\ c^h_{1,n+1}(t_s)] \tag{146}$$

$$A_4 = \sum_{s=1}^{m} [c^p_{n+1}(t_s) - c_e(t_s)]\ c^h_{1,n+1}(t_s) \tag{147}$$

$$B_1 = \sum_{s=1}^{m} [c^h_{1,n+1}(t_s)\ c^h_{2,n+1}(t_s)] \tag{148}$$

$$B_2 = \sum_{s=1}^{m} [c^h_{2,n+1}(t_s)]^2 \tag{149}$$

$$B_3 = \sum_{s=1}^{m} [c^h_{3,n+1}(t_s)\ c^h_{3,n+1}(t_s)] \tag{150}$$

$$B_4 = \sum_{s=1}^{m} [c^p_{n+1}(t_s) - c_e(t_s)]\ c^h_{2,n+1}(t_s) \tag{151}$$

$$D_1 = \sum_{s=1}^{m} [c^h_{1,n+1}(t_s)\ c^h_{3,n+1}(t_s)] \tag{152}$$

$$D_2 = \sum_{s=1}^{m} [c^h_{2,n+1}(t_s)\ c^h_{3,n+1}(t_s)] \tag{153}$$

$$D_3 = \sum_{s=1}^{m} [c^h_{3,n+1}(t_s)] \tag{154}$$

$$D_4 = \sum_{s=1}^{m} [c^p_{n+1}(t_s) - c_e(t_s)]\ c^h_{3,n+1}(t_s) \tag{155}$$

The calculations were carried out on an IBM computer 360. The computer flowchart and computer program are shown in Figure AVI - 1 and Table AVI - 2 respectively.

Furthermore, in actual calculation the initial values of δ which appear in Equations (118), (119), and (120) are set equal to a very small value (for example 10^{-7}) to avoid computer overflow. Since at initial time the term $(H_n+\delta_n)$ which appears as a denominator would be equal to zero, the value of $\delta_{n+1}(0)$ as calculated by Equation (126) is essentially unchanged by this adjustment and hence Equation (127) is still a good approximation.

Convergence Consideration of Calculation

What is really desired in the theory of differential equations, and functional equations in general, is a method of succesive

approximations which yields rapid convergence to the solution whenever the solution exists and throughout its whole domain of existence. The convergence aspect of the feedlot problem under consideration is found to be two-fold.

First, the evaluations of $C_{n+1}(0)$, $W_{n+1}(0)$ and $\delta_{n+1}(0)$ by Equations (141), (142), and (143) would cause the computer to overflow whenever the determinants appearing in the denominators approach zero. This kind of difficulty was encountered occasionally in actual calculations. To overcome this inefficiency, the integration constants $C_{n+1}(0)$, $W_{n+1}(0)$ and $\delta_{n+1}(0)$ can be determined by using an alternative method, namely, a three-dimensional pattern search technique as will be shown later. This will avoid the evaluations given by Equations (141), (142) and (143). Second, the quasilinearization technique is a second-order iterative process; convergence, if it occurs at all, is quadratic and hence rapid. However, if the initial trial solution is not sufficiently close to the true solution, then convergence may not occur with the scheme described in the preceding section. For this problem, parameters estimated through the use of the analog simulation of Table 1, which appeared in the previous section are thus helpful in guessing a initial trial solution.

Also to achieve convergence, in general, the special scheme suggested by Donnelly and Quon (16) may be used. According to their method, the total increment in going from the n-th iteration to the (n+1)st iteration might not be used but rather only a fraction. The following equations may be used to describe this method, i.e.

$$H^*_{n+1} = H_n + \xi_1(H_{n+1} - H_n) \tag{156}$$

$$C^*_{n+1} = C_n + \xi_2(C_{n+1} - C_n) \tag{157}$$

$$W^*_{n+1} = W_n + \xi_3(W_{n+1} - W_n) \tag{158}$$

$$\delta^*_{n+1} = \delta_n + \xi_4(\delta_{n+1} - \delta_n) \tag{159}$$

where $0 < \xi_i < 1.0$ $(i = 1, 2, \ldots 4)$.. Instead of using H_{n+1}, C_{n+1}, W_{n+1}, and δ_{n+1} for the next calculation, H^*_{n+1}, C^*_{n+1}, W^*_{n+1}, δ^*_{n+1} are used as the new estimated solution for next calculation. The choice of $\xi_i = 1.0$ gives rise to the original algorithm which is used previously.

This technique was used satisfactorily in recovering several sets of parameters. In fitting the experimental data 5(U), for example, the use of the original algorithm did not give rise to a convergence, while with a choice of $\xi_i = 0.5$ $(i = 1, 3, 4)$ and $\xi_2 = 0.4$, this technique gave rise to a convergence with four digital accuracy in 18 iterations. The tendency and speed of convergence also depend on the values of ξ_i used. For example, in fitting the experimental data which is noted above, a choice of $\xi_i = 0.8$ $(i = 1, 2, 3, 4)$ did not give rise to a convergence, which for a choice of $\xi_i = 0.5$ $(i = 1, 3, 4)$ and $\xi_2 = 0.4$; 18 iterations were required, and for a choice of $\xi_i = 0.3$ $(i = 1,2, 3, 4)$ 27 iterations were required to obtain a convergence with 4 digital accuracy. The choice of an optimal value of ξ_i in fitting a specific set of experimental data so as to minimize the number of iterations required is a matter of trial and error.

The Use of the Search Teachniques

A pattern search technique is usually the most straightforward approach of optimization. When the evaluation of Equations (141), (142), and (143) on a digital computer is unstable, a search technique can be employed in minimizing the functional Φ_{n+1} which is given in Equation (131).

In solving a n-dimensional optimization problem, various pattern search techniques are available, for example, the Simplex method (8), the Box Method (9), etc. The computer program used in this calculation is developed by Chen (18). This program consists of three search techniques, namely, the Simplex method, the Box method, and a modified method in which the centroid of searching points is defined in a more elaborate manner than in the original Simplex method so as to obtain a better succeeding point. The search method desired is chosen by reading in the proper constant in the main computer program. The computer program is shown in Table AVI-2.

This technique was found to be useful. All sets of parameters except 3(U) and 4(U) appeared in Table 2 were recovered by the use of this computational scheme. The additional computing time required by this modification was found to be insignificant as compared to the use of the differentiation method.

Further Considerations and Conclusions

The observed data of field study (for example, surface runoff) are usually scattered due to the complex characteristics of the system. Data fitting by the conventional "least squares"

criterion in which the same weighting factor is employed for every observed data point is not an adequate criterion for all cases. A modified "least squares" criterion which defines the minimizing functional as

$$\Phi = \sum_{i=1}^{S} \left\{ \frac{y_e(x_i) - y(x_i)}{y_e(x_i)} \right\}^2 \qquad (160)$$

is proposed by Sugie (10). Different weighting factors are now imposed on different observations. In other words, the relative deviation is taken into account instead of the absolute deviation which was considered previously.

Tables 2 and 3 show the parameters and initial concentration determined by the quasilinearization technique using two different criteria. Table 3 contains more unreasonable sets of parameters than those in Table 2. It may be concluded that the use of this modified criterion is not justifiable for this specific problem.

Furthermore, if a set of observations is very scattered, the convergence of the "best fit" solution may not exist. This is found to be true in several sets of experimental data. Irrational sets of parameters may also be obtained in fitting sets of observations with unexpected form, for example, in Table 3, the injection rates are found to be negative quantities for experimental runs 6(S) and 7(U).

The progressive iterations are sketched for several sets of data fitting. Figures 29 and 30 show the convergences of water head H and COD concentration C for 4(U) experimental run respectively. The convergences of injection rate W and weir height δ

Table 2. Parameter determined by curve fitting through the use of quasilinearization technique, with the minimization of functional

$$\Phi = \sum_{i=1}^{m} [c(t_i) - c_e(t_i)]^2$$

Experimental run	Lot condition	Rainfall intensity in./hr.	Initial concentration lbs/ft³	Injection rate lbs/min	Weir height ft	Functional Φ
1(S)	SM	0.72				
1(U)	SM	0.88	0.413508	0.612633	0.002706	0.446579×10^{-3}
2(S)	SM	0.88				
2(U)	SM	0.98	0.398916	0.062872	0.221774	0.258256×10^{-2}
3(S)	D	0.34	0.422045	0.177494	0.014196	0.479528×10^{-2}
3(U)	D	0.53	0.539052	0.641664	0.015631	
4(S)	W	0.40				
4(U)	W	0.50	0.540846	0.378140	0.071692	
5(S)	SM	0.39				
5(U)	SM	0.38	0.313937	0.214406	0.032245	0.461303×10^{-2}
6(S)	SM	0.36	0.398493	0.271741	0.036823	0.121244×10^{-1}

Table 2. (cont'd)

Experimental run	Lot condition	Rainfall intensity in./hr.	Initial concentration lbs/ft^3	Injection rate lbs/min	Weir height ft	Functional Φ
6(U)	SM	0.46				
7(S)	M	0.33	0.520206	0.174000	0.023530	0.129518×10^{-1}
7(U)	P	0.54	0.403634	-0.322698	0.242300	0.621116×10^{-2}
8(S)	M	0.43	0.559015	0.396602	0.017701	0.719299×10^{-2}
8(U)	P	0.59	0.353835	0.369464	0.010669	0.416427×10^{-2}
9(S)	M	0.63	0.359280	0.147009	0.050830	0.681253×10^{-2}
9(U)	P	0.42				
10(S)	M	0.58	0.563164	0.445334	0.010354	0.281037×10^{-1}
10(U)	P	0.22				

Table 3. Parameter determined by curve fitting through the use of quasilinearization technique, with the minimization of functional

$$\Phi = \sum_{i=1}^{m} \left\{ \frac{C(t_i) - C_e(t_i)}{C_e(t_i)} \right\}^2$$

Experimental run	Lot condition	Rainfall intensity in./hr.	Initial concentration lbs/ft^3	Injection rate lbs/min	Weir height ft	Functional Φ
1(S)	SM	0.72				
1(U)	SM	0.88	0.311700	-1.254822	0.911251	0.955687×10^{-1}
2(S)	SM	0.88				
2(U)	SM	0.98	0.399724	0.271781	0.154916	0.372926×10^{-1}
3(S)	D	0.34	0.411948	0.194132	0.011610	0.331571×10^{-1}
3(U)	D	0.53	0.532575	0.649382	0.013851	0.245803×10^{-1}
4(S)	W	0.40				
4(U)	W	0.50	0.539898	0.390393	0.067937	0.479149×10^{-2}
5(S)	SM	0.39				
5(U)	SM	0.38	0.317916	0.222539	0.027159	0.523287×10^{-1}
6(S)	SM	0.36	0.390963	-0.638889	0.593131	0.747795×10^{-1}

Table 3. (cont'd)

Experimental run	Lot condition	Rainfall intensity in./hr.	Initial concentration lbs/ft^3	Injection rate lbs/min	Weir height ft	Functional Φ
6(U)	SM	0.46				
7(S)	M	0.33	0.505040	0.206031	0.019231	0.441891×10^{-1}
7(U)	P	0.54	0.393917	-1.137894	0.541948	0.554176×10^{-1}
8(S)	M	0.43	0.549012	0.413161	0.015299	0.302936×10^{-1}
8(U)	P	0.59	0.340503	0.385989	0.008154	0.482383×10^{-1}
9(S)	M	0.63	0.341742	0.172991	0.051414	0.642090×10^{-1}
9(U)	P	0,42				
10(S)	M	0.58	0.505990	0.481670	0.013002	0.134748×10^{0}
10(U)	P	0.22				

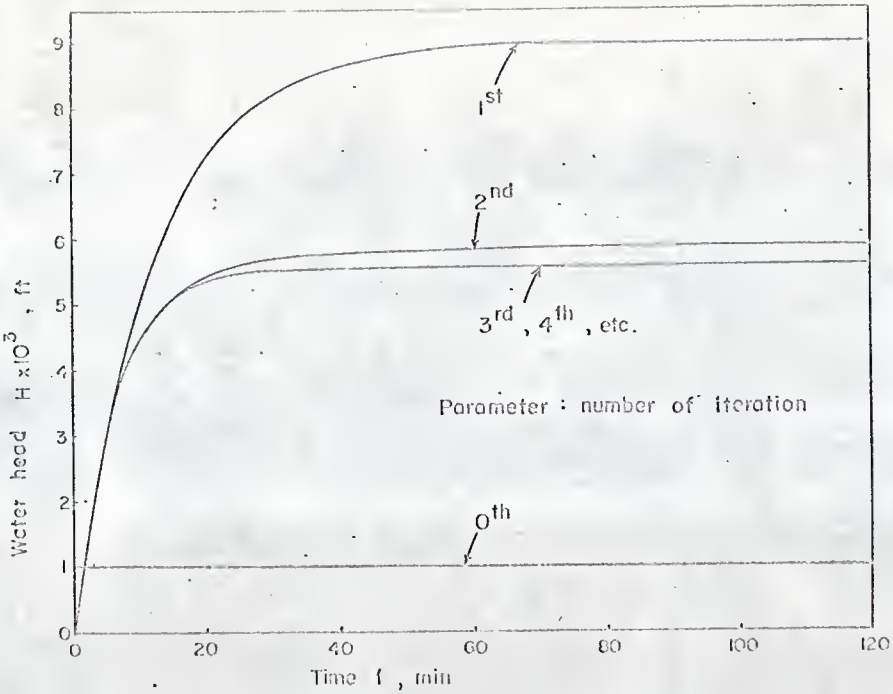


Fig.29. Convergence of water head H for 4 (U) experimental run.

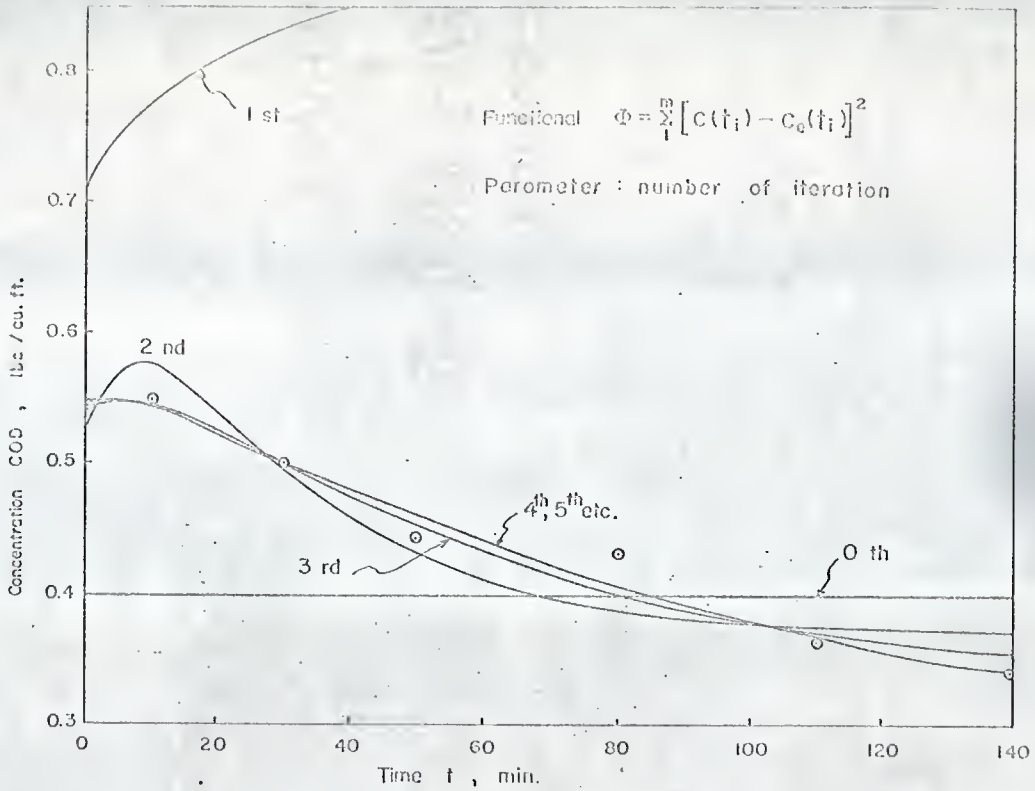


Fig.30. Curve fitting of experimental data by quasilinearization for 4(U) experimental run.

are tabulated in Table 4. Convergences of data fitting for 8(S) and 8(U) experimental runs are shown in Figures 31 and 32 and Table 5, and Figures 33 and 34 and Table 6 respectively. The convergence of water head H was obtained in fewer iterations (about 4) than those required for the concentration of COD.

Table 4. Convergence of injection rate W and weir height δ for 4(U) experimental run

Number of iteration	Injection rate W, lbs/min	Weir height δ, ft
0	0.3000	0.04000
1	0.7857	0.03571
2	0.5666	0.01976
3	0.4872	0.03552
4	0.4348	0.05394
5	0.3945	0.06685
6	0.3803	0.07113
7	0.3783	0.07166
8	0.3781	0.07169

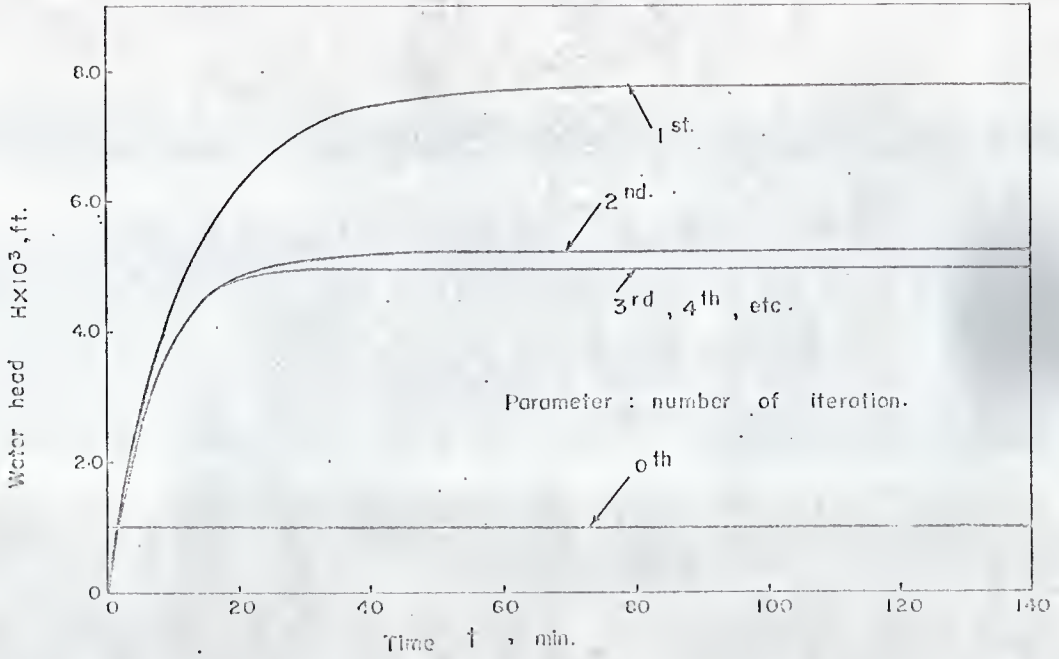


Fig. 31 Convergence of water head H for 8(S) experimental run.

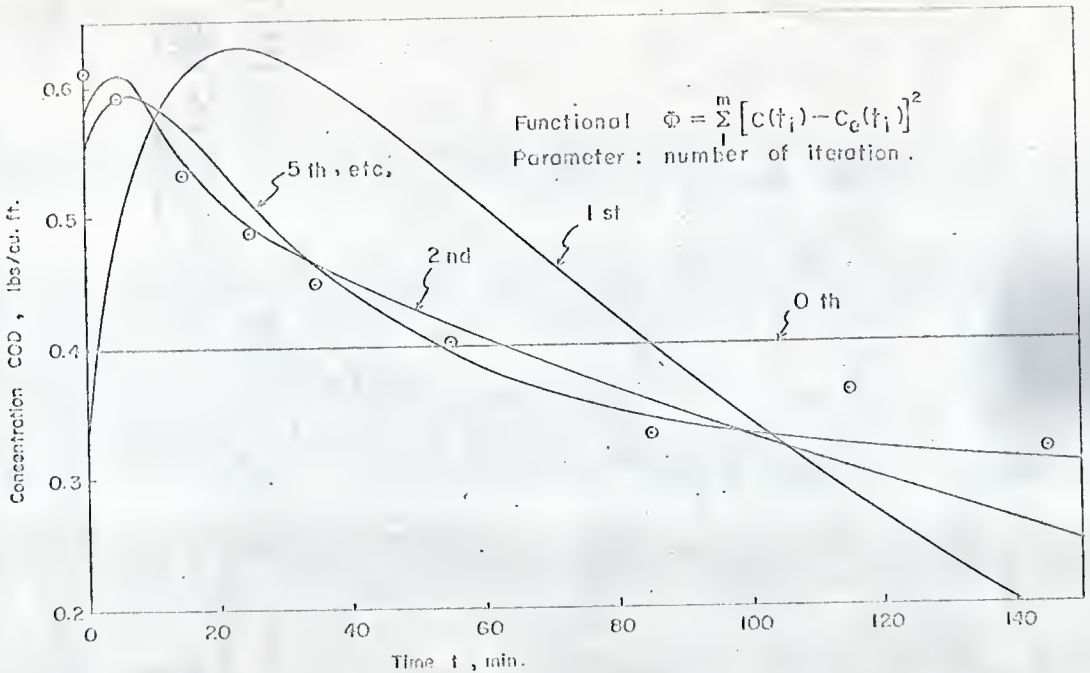


Fig.32. Curve fitting of experimental data by quasilinearization for S(s) experimental run.

Table 5. Convergence of injection rate W and weir height δ for 8(S) experimental run

Number of iteration	Injection rate W, lbs/min	Weir height δ, ft
0	0.400000	0.009000
1	0.523190	0.004709
2	0.379049	0.007031
3	0.369639	0.013073
4	0.383846	0.017540
5	0.396602	0.017701
6	0.396602	0.017701

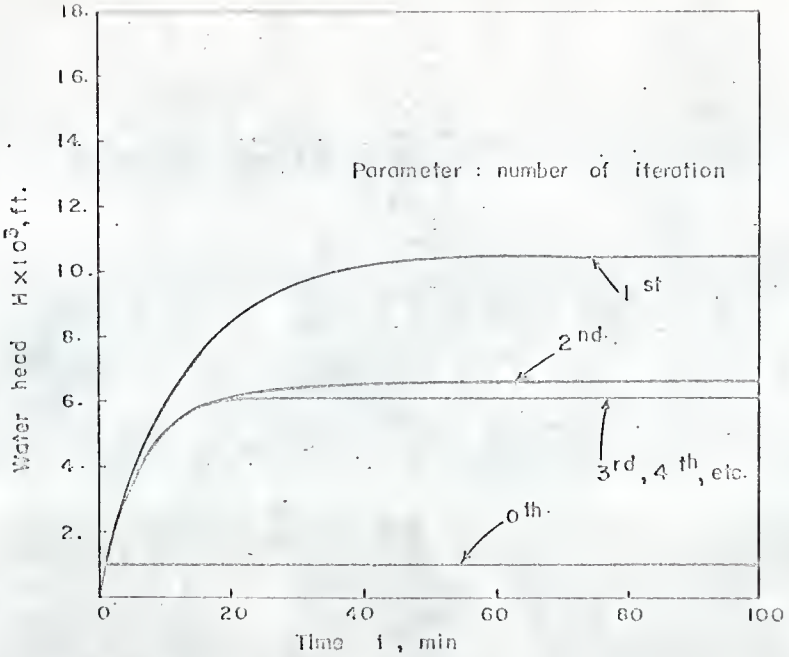


Fig. 33. Convergence of water head H for 8(U) experimental run.

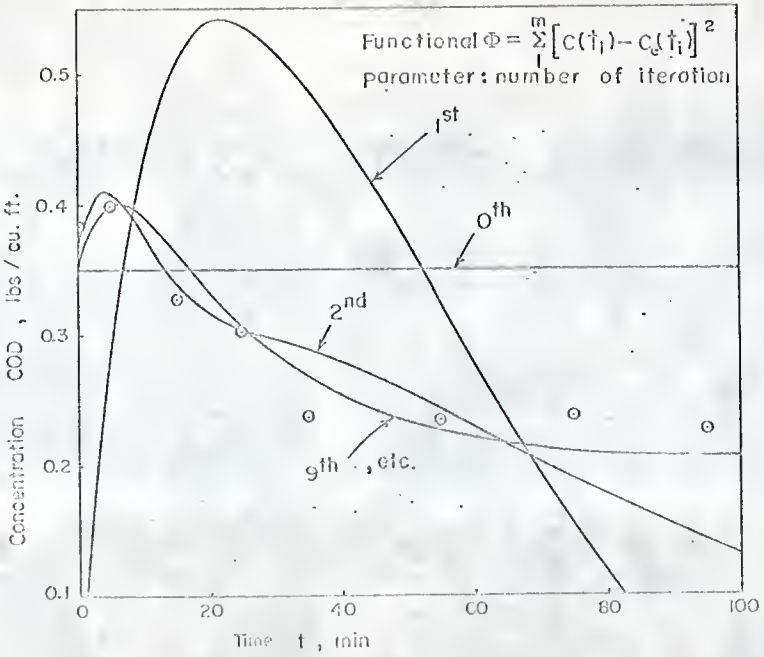


Fig.3.4. Curve fitting of experimental data by quasilinearization for 8(U) experimental run.

Table 6. Convergence of injection rate W and weir height δ for 8(U) experimental run

Number of iteration	Injection rate W, lbs/min	Weir height δ, ft
0	0.200000	0.005000
1	0.673847	0.001921
2	0.344508	0.002656
3	0.366133	0.005685
4	0.363183	0.009912
5	0.367367	0.010950
6	0.369096	0.010707
7	0.369805	0.010493
8	0.369464	0.010669
9	0.369464	0.010669

CONCLUSION AND RECOMMENDATIONS

The first phase study of stream pollution by cattle feedlot has been presented. By virtue of the complexity of the physical system, it is difficult to make decisive conclusions on this problem. However, according to this study which has attempted to predict the gross behavior of the system, several remarks can be made.

(1) Consideration of the dynamic characteristic of the system by using the Stirred Tank with Injection model seems justifiable. This consideration gives rise to the characteristic initial increase of COD concentration which can only be obtained by employing the Series Stirred Tank with Injection model, as was shown by Miner (1). The use of the latter model would introduce five parameters (rather than three) in the system. These will complicate the system analysis and simulation study. Furthermore, use of the Series Stirred Tank with Injection model together with the consideration of the dynamic characteristic of the system would require seven nonlinear operators on an analog computer in simulation study. These can not be afforded by a compact-sized analog computer in some cases.

(2) The outflow rate Q as function of time is an important characteristic of the dynamic study of the system. Additional observations on outflow rate Q from the feedlot may add information to conduct a better investigation. It is suggested that more experimental data of flow rate from feedlot with respect to time be taken.

(3) Despite the various surface conditions of the feedlot system, the injection rate W under different cases is found to have an approximately linear relation with rainfall intensity. The correlations have been shown in Figures 9, 10, and 11. This discovery may largely simplify the analysis and the management of the pollution problem in many aspects.

(4) A sufficient number of observations are required to conduct a concrete investigation of the system. The experimental data available in this study are limited. Several sets of experimental data of COD concentration versus time are found to have too much scatter to use them in the quasilinearization technique of parameter recovery. Furthermore, most sets of data were taken in a large time interval (10 - 30 min.). This is considered as an inefficiency in making a proper fit by analog simulation. More data taken in a short time interval are thus desirable.

(5) For a system of three parameters, fitting data by "twiddling" the knobs in an analog computer may be used easily when too much accuracy is not required. By this method convergence can almost be assured even if the data are scattered. The use of quasilinearization technique is more general and accurate in a sense. When the convergence of the solution occurs, that is, when the experimental data are in good agreement with the proposed model, this technique is very effective. The convergence of the solution by quasilinearization technique is partially dependent on the initial trial solution. The pre-study of analog simulation is thus helpful in making a proper guess.

ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to Dr. Liang-tseng Fan and Dr. Larry E. Erickson for their constant advice and enthusiasm; Dr. William H. Honstead for his help and encouragement; Dr. Richard G. Akins for reading the manuscript. The author would also like to acknowledge the Agricultural Experimental Station of Kansas State University for financial support and the Kansas State University Computing Center for the use of the Computer.

APPENDICES

APPENDIX I. THE RUNGE-KUTTA METHOD OF TWO SIMULTANEOUS EQUATIONS (17)

The Runge-Kutta method for numerical integration has been described by Runge (22) and elaborated by Kutta (23). In this method, formulas are devised that enable the direct calculation of the increment in dependent variable y corresponding to an increment in independent variable x. To integrate numerically higher order equation or simultaneous equations by this method, essentially the same method is used as that used for a single equation.

Consider the following set of simultaneous equations.

$$\frac{dy}{dx} = f_1(x, y, z) \tag{1}$$

$$\frac{dz}{dx} = f_2(x, y, z) \tag{2}$$

Starting at the initial values x_o, y_o, z_o, the increments in y and z for the first increment in x are computed by means of the formulas

$$k_1 = f_1(x_o, y_o, z_o)\Delta x \tag{3}$$

$$l_1 = f_2(x_o, y_o, x_o)\Delta x \tag{4}$$

$$k_2 = f_1(x_o + \frac{\Delta x}{2}, y_o + \frac{k_1}{2}, z_o + \frac{l_1}{2})\Delta x \tag{5}$$

$$l_2 = f_2(x_o + \frac{\Delta x}{2}, y_o + \frac{k_1}{2}, z_o + \frac{l_1}{2})\Delta x \tag{6}$$

$$k_3 = f_1(x_o + \frac{\Delta x}{2}, y_o + \frac{k_2}{2}, z_o + \frac{l_2}{2})\Delta x \tag{7}$$

$$l_3 = f_2(x_o + \frac{\Delta x}{2}, y_o + \frac{k_2}{2}, z_o + \frac{l_2}{2})\Delta x \tag{8}$$

$$k_4 = f_1(x_o + \frac{\Delta x}{2},\ y_o + \frac{k_3}{2},\ z_o + \frac{l_3}{2})\,\Delta x \tag{9}$$

$$l_4 = f_2(x_o + \frac{\Delta x}{2},\ y_o + \frac{k_3}{2},\ z_o + \frac{l_3}{2})\,\Delta x \tag{10}$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{11}$$

$$\Delta z = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \tag{12}$$

thus,

$$x_1 = x_o + \Delta x \tag{13}$$

$$y_1 = y_o + \Delta y \tag{14}$$

$$z_1 = z_o + \Delta z \tag{15}$$

To compute the next increment, it is necessary only to replace x_o, y_o, and z_o in the above formulas by x_1, y_1, and z_1 and continue this procedure.

The procedure is continued until the desired final values of the dependent variables y and z are obtained. A choice of smaller increment Δx will give a solution with better accuracy.

APPENDIX II. RANDOM NUMBER GENERATION (11)

1. Introduction

There are many ways to generate a set of random numbers. The principal requirement of a sequence of random number is statistical independence. Four alternative methods have been used by practitioners to generate sequences of random numbers. They are

1. Manual methods.
2. Library tables.
3. Analog computer methods.
4. Digital computer methods.

Manual methods are the simplest and also the least practicable of the methods for generating random numbers. These methods, are usually too slow for general use. Another disadvantage is that it is impossible to reproduce a sequence of random numbers generated by such devices.

A number of Library Tables of random numbers (for example, the Rand Corporation's A Million Random Digits) have been published. Of course, these numbers must first be generated by one of the aforementioned methods before recording them in table form. The advantage offered by this method is reproducibility. However, it suffers from a lack of speed and the limited length of the random numbers sequence.

Analog computers have also been used to generate random numbers. They are deemed to yield "truly" random numbers, since

analog computer methods depend on some random physical process. These methods are much faster than either manual methods or library tables, but share a common handicap with manual methods-nonreproducible sequences of random numbers.

Three modes of providing random numbers for use on digital computers have been suggested by Tocher (24). They are (1) external provision, (2) internal generation by a random physical process, and (3) internal generation of sequences of digits by a recurrence relation. The third method is the most satisfactory way of generating a sequence of "pseudorandom numbers". The term "pseudorandom numbers" has been defined by Lehmer (25) as "a vague notion embodying the idea of a sequence in which each term is unpredictable to the uninitiated and whose digits pass a certain number of tests, traditional with statisticians and depending somewhat on the use to which the sequence is to be put". Tocher (26) has stated that, "the principal objection to this solution is on the rather philosophical grounds that a sequence of digits generated by a pure deterministic rule is the direct antithesis of a random sequence". However, this objection can at least partially be overcome by taking the pragmatic view that a sequence may be considered random if it satisfies some predetermined set of statistical tests of randomness. From this point of view the method of generating a sequence is totally irrelevant.

An "acceptable" method for generating random numbers must yield sequences of numbers which are (1) uniformly distributed, (2) statistically independent, (3) reproducible, and (4) nonrepeating for any desired length. Furthermore, such a method must also be

capable of (5) generating random number at high rates of speed, and yet (6) requiring a minimum amount of computer memory capacity. The congruential methods to be discussed were designed specifically to fulfill as many of the aforementioned requirements as possible.

2. Congruential methods for generating pseudorandom numbers

The congruential methods for generating random numbers are completely deterministic because the arithmetic process involved in the calculations uniquely determines each term in a sequence of numbers. In fact, formulas are available for calculating in advance the exact value of the i-th number in a sequence of numbers n_0, n_1, n_2, ..., n_i, ... before the sequence is actually generated.

Properties (3) and (6) of the afore-mentioned requirements of random number generators are automatically satisfied by the application of congruential methods because the sequences generated by these methods are completely reproducible and require only a minimum amount of memory capacity on a computer. Properties (4) and (5) are the only requirements whose degree of satisfaction depends entirely on the properties of the methods applied.

Congruential methods are based on a fundamental congruence relationship, which may be expressed as the following recursive formula

$$n_{i+1} \equiv a\, n_i + c \pmod{m}^* \qquad (1)$$

---

* By definition, two integers a and b are congruent modulo m if their difference is an integral multiple of m. The congruence relation is expressed by the notation $a \equiv b \pmod{m}$, which reads "a is congruent b modulo m"; this also means that (1) (a - b) is divisible by m and (2) a and b leave identical

where n_1, a, c, and m are all nonnegative intergers. Writing Equation (1) for i = 0, 1, 2, ..., we obtain

$$
\begin{aligned}
n_1 &\equiv an_o + c \pmod m \\
n_2 &\equiv an_1 + c \equiv a^2 n_o + (a + 1)\, c \pmod m \\
n_3 &\equiv a^3 n_o + (a^2 + a + 1)\, c \equiv a^3 n_o + \frac{c(a^3 - 1)}{(a - 1)} \pmod m \qquad (2) \\
&\text{- - - - - - - - - - - - - -} \\
&\text{- - - - - - - - - - - - - -} \\
&\text{- - - - - - - - - - - - - -} \\
n_i &\equiv a^i n_o + \frac{c(a^i - 1)}{(a - 1)} \pmod m
\end{aligned}
$$

Given an initial starting value n_o, a constant multiplier a, and an additive constant c, then Equation (2) yields a congruence relationship (modulo m) for any value of i over the sequence $\{n_1, n_2, \ldots, n_i, \ldots\}$. The subsequent terms of $\{n_i\}$ as determined by Equation (2) are all integers forming a sequence of residues modulo** m. This implies that $n_i < m$ for all n_i. Now, the question is what conditions can be imposed on n_o, a, c, and m so that the period of $\{n_i\}$ is as large as possible ? By number theory it can be shown that it is impossible to obtain nonrepeating sequences by congruential methods. In practice, however, the period of a sequence can be set satisfactorily high by choosing a sufficiently large modulus or by other techniques (27).

3. The Multiplicative Congruential Method

---

remainders when divided by m. Example: $5590 \equiv 6 \equiv -2 \pmod 8$ and $2327 \equiv 27 \pmod{10^2}$

** For a given a the smallest positive integer n such that $a \equiv n \pmod m$ is said to be a residue modulo m. There are m distinct residues (mod m); 0, 1, 2, ..., m-1.

Most computerized versions of the multiplicative congruential method employ a modulus $m = p^e$, representing the word size of the computer, where p denotes the number of numerals in the number system utilized by the computer and e denotes the number of digits in a word, for binary computers $p = 2$, and for decimal computers $p = 10$. The formula for generating power residues is (by setting $c = 0$ in Equation (1))

$$n_{i+1} \equiv an_i \pmod{p^e} \tag{3}$$

For a binary computer (for example, an IBM computer 3-60) we choose $m = 2^b$, where b is the number of binary digits(bits) in a word. According to the number theory (11), the maximum attainable period is $h = 2^{b-2}$. The next problem is to find constant multipliers that have order $h = 2^{b-2}$. It can also be shown by number theory that a must be relatively prime to m. Furthermore, if a is relatively prime to $m = 2^b$, it must be an odd number. It can be shown that those values of a, which satisfy these requirements, reside in a residue class represented by the congruence relation (28)

$$a \equiv \pm 3 \pmod{8} \tag{4}$$

This relation can also be written as

$$a = 8t \pm 3 \tag{5}$$

where t is any positive integer.

According to Greenberger's formula, values of a that are close to $2^{b/2}$ will minimize first-order serial correlation between the

pseudorandom numbers. Since small serial correlation is a highly desirable characteristic of random number generators, this rule will be applied in determining the "best" constant multipliers for both the binary and decimal cases. Having selected a constant multiplier, we must now consider a method for choosing a starting value. According to number theory, n_o must be relatively prime to 2. This requirement can be satisfied by selecting any positive odd number for a starting value.

The multiplicative procedure for generating random numbers on a binary machine may be summarized as follows:

(1) Choose any odd number as a starting value.

(2) Choose an integer $a = 8t \pm 3$, where t is any positive integer for a constant multiplier. A value of a close to $2^{b/2}$ will satisfy the Coveyou-Greenberger condition. (for an IBM computer 3-60, b = 31, thus $a = 2^{16} + 3 = 65539$ is a good selection).

(3) Compute a n_o using fixed point integer arithmetic. This product will consist of 2b bits, from which the higher-order b bits are discarded, and the low order b bits represent n_1. (The integer multiplication instruction in FORTRAN automatically discards the high-order b bits).

(4) Calculate $Y_1 = n_1/2^b$ to obtain a uniformly distributed variate defined on the unit interval.

(5) Each successive random number n_{i+1} is obtained from the low-order bits of the product an_i.

The statistical properties of pseudorandom numbers generated by this deterministic method are not random in the sense. However,

so long as our pseudorandom numbers can pass the set of statistical tests implied by the idealized chance device, these pseudorandom numbers can be treated as "truly" random numbers even though they are not. The chi-square test are most general method for test of randomness. It has been shown (11) that the pseudorandom numbers generated by this manner passed satisfactorily the statistical tests.

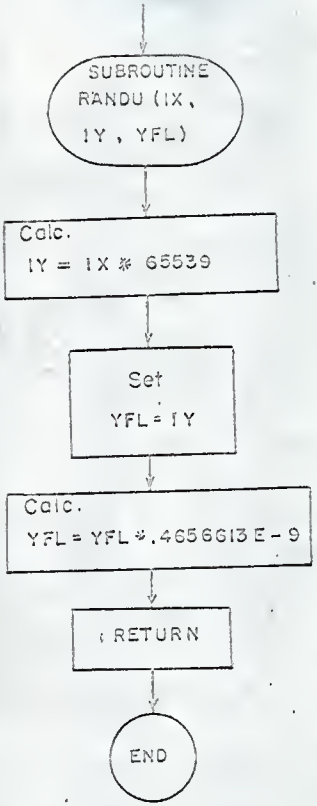


Fig. A II-1. Computer flow chart for generating uniform random numbers between zero and one.

APPENDIX III. RANDOM NUMBER WITH UNIFORM DISTRIBUTION

The desired uniform distribution with specified mean $\frac{A + B}{2}$ can be generated by the relationship

$$X = A + (B - A) \times R$$

where R = random variable between 0 and 1

A = specified upper range of desired distribution

B = specified lower range of desired distribution

X = desired distribution.

The technique of generating random variable R was described in Appendix II.

The computer flow chart for generating a random numbers with uniform distribution with mean $\frac{A + B}{2}$ is show in Figure AIII-1.

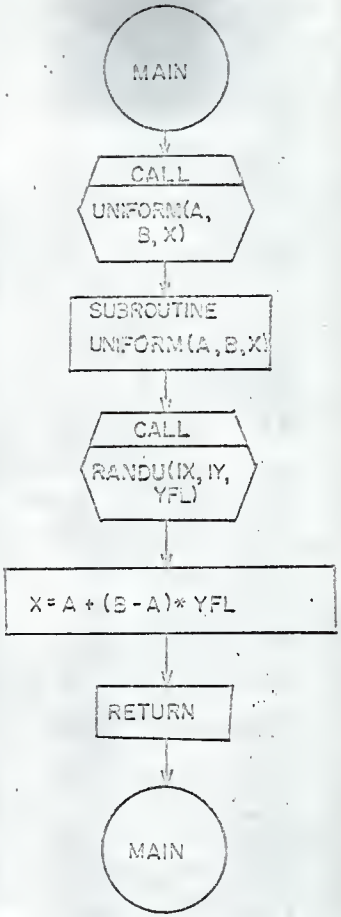


Fig. A III-1 Flow chart for generation of uniformly distributed random variates with mean $\frac{a+b}{2}$.

APPENDIX IV. RANDOM VARIABLE WITH NORMAL DISTRIBUTION

Random variates with normal distribution can be generated from a set of uniform distributed random variates. According to the central limiting theorem (15), the random variable of a normal distribution with mean zero and variance one can be obtained from a set of random sample of uniform distribution between zero and one. The equation which relates these two random variables can be written as

$$Y = \frac{\sum_{1}^{k} X_i - \frac{1}{2}k}{\sqrt{k/12}}$$

where Y has a normal limiting distribution with mean of zero and variance of one, X_i is the random sample from a uniform distribution, and k is the number of values of X_i to be used. Y approaches a true normal distribution if k approaches infinity.

The desired random sample of a normal distribution with specified mean and variance can be obtained as

$$Y' = Y\sigma + \mu$$

where σ = specified standard deviation

μ = specified mean

Y' = desired random variable.

The technique of generating a uniform distributed random variable has been discussed in the preceding two sections. The flow chart for a computing scheme for generating a random variate

with normal distribution is shown in Figure A V.I - 1.

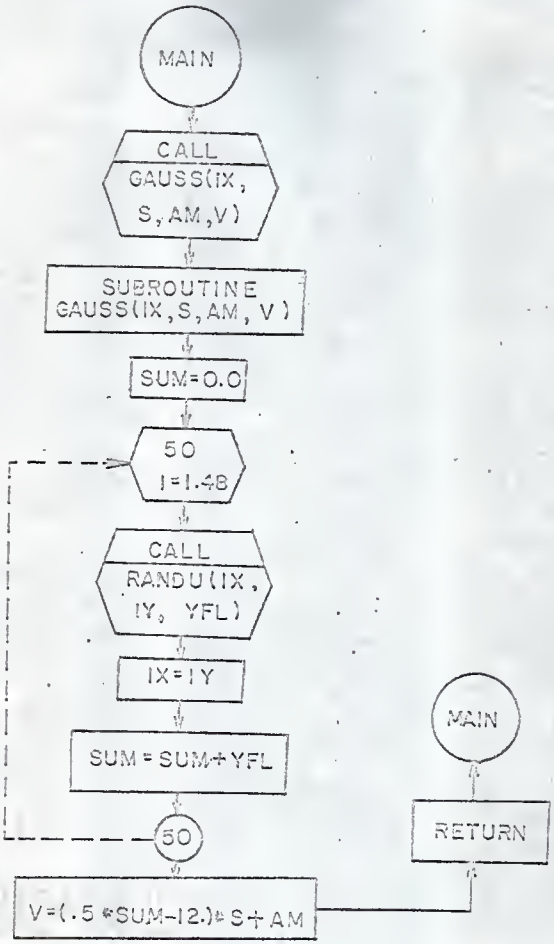


Fig.AⅢ-1. Flow chart for generation of normal variates.

APPENDIX V. COMPUTATION IN RANDOM SIMULATION

The computer flowchart and FORTRAN computer program for random simulation are presented in Figure AV - 1 and Table AV - 2 respectively. The nonlinear system equations are solved by using the Runge-Kutta numerical integration method.

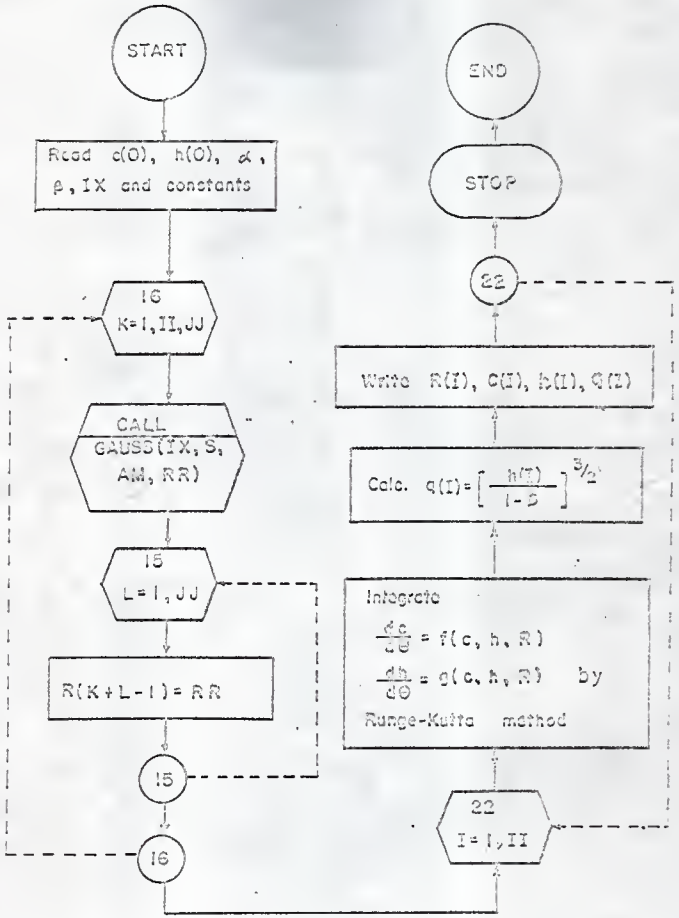


Fig. AV-1. Computer flowchart for random simulation.

Table AV - 1. Program Symbols and Explanation

Program Symbols	Explanation	Mathematical Symbols
A	dimensionless parameters	α
AM	mean of random variables with normal distribution	μ
B	dimensionless parameter	β
C_1	initial concentration	c_o
C(I)	COD concentration	C(t)
DC	increment of C(I)	Δc
DH	increment of H(I)	Δh
DT	increment of θ	$\Delta \theta$
H(I)	water head	h(t)
II	constant	
IX	initial value used in generating uniform random variate	n_o
JJ	constant	
Q(I)	flow rate	q(t)
R(i)	rainfall intensity	r(t)
RR	normal random variate	y'
S	standard deviation of random variables with normal distribution	σ
T	time	θ

Table AV - 2. Computer program for random simulation

```
      DIMENSION C(100),H(100),Q(100),R(100)
  101 FORMAT(1F10.2,1F10.3,3F10.8)
  102 FORMAT(2I5,1F10.2)
  103 FORMAT(3F10.2)
  104 FORMAT(///,7X,2HT=,6X,2HR=,6X,4HCOD=,6X,2HH=,6X,2HC=,/)
      READ(1,102)II,JJ,DT
      WRITE(3,102)II,JJ,DT
      READ(1,103)C1,A,B
      WRITE(3,103)C1,A,B
      IX=7654321
      S=.5
      AM=1.0
      H(1)=0.
      C(1)=C1
      WRITE(3,104)
      DO16K=1,II,JJ
      CALL GAUSS(IX,S,AM,RR)
      DO15L=1,JJ
   15 R(K+L-1)=RR
   16 CONTINUE
      DO22I=1,II
      FK1=(R(I)-(H(I)**1.5)/(1.-B)**1.5)*DT
      FL1=((R(I)-A-(C(I)*H(I)**1.5)/((1.-B)**1.5))/(H(I)+B))*DT
      H2=H(I)+FK1/2.
      C2=C(I)+FL2/2.
      FK2=(R(I)-H2**1.5/(1.-B)**1.5)*DT
      FL2=((R(I)-A-(C2*(H2)**1.5)/((1.-B)**1.5))/(H2+B))*DT
      H3=H(I)+FK2/2.
      C3=C(I)+FL2/2.
      FK3=(R(I)-H3**1.5/(1.-B)**1.5)*DT
      FL3=((R(I)-A-(C3*(H3)**1.5)/((1.-B)**1.5))/(H3+B))*DT
      H4=H(I)+FK3/2.
      C4=C(I)+FL3/2.
      FK4=(R(I)-H4**1.5/(1.-B)**1.5)*DT
      FL4=((R(I)-A-(C4*(H4)**1.5)/((1.-B)**1.5))/(H4+B))*DT
      DH=(FK1+2.*(FK2+FK3)+FK4)/6.
      DC=(FL1+2.*(FL2+FL3)+FL4)/6.
      Q(I)=(H(I)/(1.-B))**1.5
      H(I+1)=H(I)+DH
      C(I+1)=C(I)+DC
      II=I
      T=(I-1.)*DT
   22 WRITE(3,101)T,R(I),C(I),H(I),Q(I)
      STOP
      END
```

Table AV - 2. (cont'd)

```
      SUBROUTINE GAUSS(IX,S,AM,V)
      AA=0.0
      DO 50 I=1,48
      CALL RANDU(IX,IY,YFL)
      IX=IY
   50 AA=AA+YFL
      V=(.5*AA-12.0)*S+AM
      RETURN
      END


      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END
```

APPENDIX VI. COMPUTATION OF PARAMETERS DETERMINATION

The computer flowchart and FORTRAN computer program for the computation of parameter determination by using quasilinearization technique are shown. Figure AVI - 1 shows the computation scheme of this technique. The quasilinearization technique with differentiation method is shown in Table AVI - 2. The quasilinearization technique with the use of search technique is shown in Table AVI - 3.

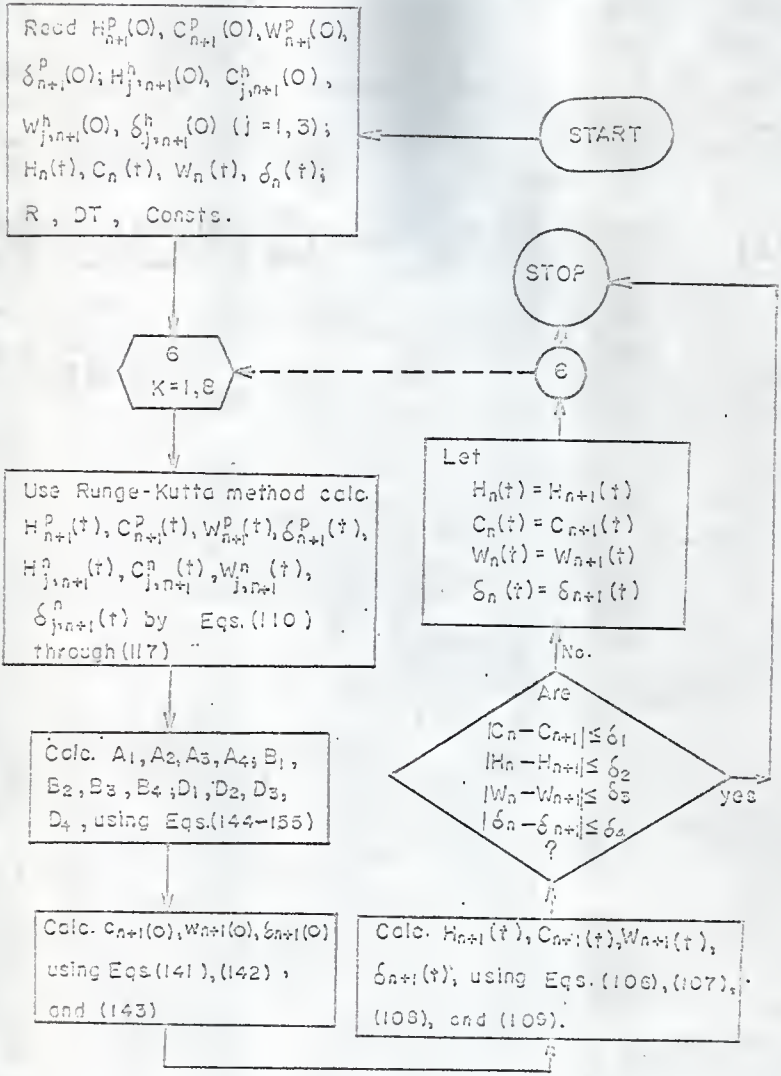

START
Read $H^p_{n+1}(0)$, $C^p_{n+1}(0)$, $W^p_{n+1}(0)$, $\delta^p_{n+1}(0)$; $H^h_{j,n+1}(0)$, $C^h_{j,n+1}(0)$, $W^h_{j,n+1}(0)$, $\delta^h_{j,n+1}(0)$ (j = 1, 3); $H_n(t)$, $C_n(t)$, $W_n(t)$, $\delta_n(t)$; R, DT, Consts.
6
K=1,8
Use Runge-Kutta method calc. $H^p_{n+1}(t)$, $C^p_{n+1}(t)$, $W^p_{n+1}(t)$, $\delta^p_{n+1}(t)$, $H^h_{j,n+1}(t)$, $C^h_{j,n+1}(t)$, $W^h_{j,n+1}(t)$, $\delta^h_{j,n+1}(t)$ by Eqs. (110) through (117)
Calc. $A_1, A_2, A_3, A_4; B_1, B_2, B_3, B_4; D_1, D_2, D_3, D_4$, using Eqs. (144–155)
Calc. $C_{n+1}(0), W_{n+1}(0), \delta_{n+1}(0)$ using Eqs. (141), (142), and (143)
8
Calc. $H_{n+1}(t)$, $C_{n+1}(t)$, $W_{n+1}(t)$, $\delta_{n+1}(t)$, using Eqs. (106), (107), (108), and (109).
Are $|C_n - C_{n+1}| \le \delta_1$, $|H_n - H_{n+1}| \le \delta_2$, $|W_n - W_{n+1}| \le \delta_3$, $|\delta_n - \delta_{n+1}| \le \delta_4$?
No.
yes
Let $H_n(t) = H_{n+1}(t)$, $C_n(t) = C_{n+1}(t)$, $W_n(t) = W_{n+1}(t)$, $\delta_n(t) = \delta_{n+1}(t)$
STOP

Fig. AVI-1 Computer flowchart for quasilinearization technique.

Table AVI - 1. Program Symbols and Explanation

Program Symbols	Explanation	Mathematical Symbols
A5 - A8	functionals defined by Equations (144) through (147)	$A_1 - A_4$
B5 - B8	functionals defined by Equations (148) through (151)	$B_1 - B_4$
CEXP(I)	experimental data of COD	$C_e(t_i)$
CII	initial trial solution of COD	$C_o(t)$
C(J, 1)	particular solution of COD	$C^p_{n+1}(t)$
C(J, L) [L = 2, 3, 4]	j th homogeneous solution of COD	$C^h_{j,\ n+1}(t)$
CODO	initial concentration of COD	$C_{n+1}(0)$
CP(J, L)	approximate solution of COD used for calculation	$C_n(t)$
CSOL(J)	new approximated solution of COD	$C_{n+1}(t)$
D5 - D8	functionals defined by Equations (152) through (155)	$D_1 - D_4$
DC	increment of COD in numerical integration	ΔC
DDEL	increment of δ in numerical integration	$\Delta\delta$
DELII	initial trial solution of weir height	$\delta_o(t)$
DEL(I, 1)	particular solution of weir height	$\delta^p_{n+1}(t)$
DEL(J, L) [L = 2, 3, 4]	j th homogeneous solution of weir height	$\delta^h_{j,\ n+1}(t)$

Table AVI - 1. (cont'd)

Program Symbols	Explanation	Mathematical Symbols
DELO	initial value of weir height	$\delta_{n+1}(0)$
DELP(J, L)	approximate solution of weir height used for calculation	$\delta_n(t)$
DELSOL(J)	new approximate solution of weir height	$\delta_{n+1}(t)$
DH	increment of H in numerical integration	ΔH
DT	increment of time in numerical integration	Δt
DW	increment of W in numerical integration	Δw
ERR1	constant	δ_1
ERR2	constant	δ_2
ERR3	constant	δ_3
ERR4	constant	δ_4
HII	initial trical solution of water head	$H_o(t)$
H(J, 1)	particular solution of water head	$H^p_{n+1}(t)$
H(J, L) [L = 2, 3, 4]	j th homogeneous solution of water head	$H^h_{j,n+1}(t)$
HP(J, L)	approximate solution of water head used for calculation	$H_n(t)$
HSOL(J)	new approximate solution of water head	$H_{n+1}(t)$
I, JJ, K, KI	constants	

Table AVI - 1. (cont'd)

Program Symbols	Explanation	Mathematical Symbols
LL, MM, NN	constants	
R	rainfall intensity	R
T	time	t
WII	initial trial solution of injection rate	$W_o(t)$
W(J, 1)	particular solution of injection rate	$W^p_{n+1}(t)$
W(J, L) L = 2, 3, 4	jth homogeneous solution of injection rate	$W^h_{j,n+1}(t)$
WO	initial value of injection rate	$W_{n+1}(0)$
WP(J, L)	approximate solution of injection rate used for calculation	$W_n(t)$
WSOL(J)	new approximate solution of injection rate	$W_{n+1}(t)$

Table AVI - 2. Computer program for quasilinearization technique

```
      DIMENSION HP(350,4),H(350,4),CP(350,4),C(350,4),WP(350,4),W
     1(350,4),DELP(350,4),DEL(350,4),HIP(4),CIP(4),WIP(4),DELIP(4)
     2,CEXP(12),CC(12,4),A5(12),A6(12),A7(12),A8(12),B5(12),B6(12)
     3,B7(12),B8(12),D5(12),D6(12),D7(12),D8(12),HSOL(350),CSOL(35
     40),WSOL(350),DELSOL(350)
  100 FORMAT(9I5)
  101 FORMAT(4F10.7)
  102 FORMAT(//,2X,'0HITERATION=',I3,/)
  103 FORMAT(2I5,F8.1,4F12.6)
  104 FORMAT(1F10.3)
  105 FORMAT(2X,2HIT,3X,3HNO.,4X,2HT=,6X,4HCOD=,8X,5HHPIJ=,7X,2HW
     1=,8X,6HDELTA=,/)
  106 FORMAT(4F10.6)
  108 FORMAT(1F5.1,1F11.6)
      READ(1,100)NN,KI,LL,JJ,MM
      WRITE(3,100)NN,KI,LL,JJ,MM
      DO11I=1,4
      READ(1,101)HIP(I),CIP(I),WIP(I),DELIP(I)
    1 WRITE(3,101)HIP(I),CIP(I),WIP(I),DELIP(I)
      READ(1,101)HII,CII,WII,DELII
      WRITE(3,101)HII,CII,WII,DELII
      READ(1,106)DT,R
      WRITE(3,106)DT,R
      READ(1,106)ERR1,ERR2,ERR3,ERR4
      WRITE(3,106)ERR1,ERR2,ERR3,ERR4
      DO22M=1,MM
      READ(1,104)CEXP(M)
   22 WRITE(3,104)CEXP(M)
      DO2L=1,LL
      DO2J=1,JJ
      HP(J,L)=HII
      CP(J,L)=CII
      WP(J,L)=WII
    2 DELP(J,L)=DELII
      DO3I=1,4
      H(J,I)=HIP(I)
      C(J,I)=CIP(I)
      W(J,I)=WIP(I)
    3 DEL(J,I)=DELIP(I)
      DO6K=1,KI
      WRITE(3,102)K
      DO5L=1,LL
      DO5J=1,NN
      HP(J,L)=HSOL(J,L)
      CP(J,L)=CSOL(J,L)
```

Table AVI - 2. (cont'd)

```
      WPIJ=WP(J,L)
      DELPIJ=DEL(J,L)
      IF(L.GE.2)GO TO 33
      PART1=-.545*(WPIJ)**1.5
      PART2=(.545*CPIJ*(WPIJ)**1.5+.000452*WPIJ)/(WPIJ+DELPIJ)
      GOTO4
   33 PART1=0.
      PART2=0.
    4 FK1=(PART1-2.540*H(J,L)*(WPIJ)**.5)*DT
      P1=(-1.69*(WPIJ)**1.5)/(WPIJ+DELPIJ)
      P2=(2.54*CPIJ*(WPIJ)**.5)/(WPIJ+DELPIJ)+(.000452*WPIJ-1.69*
     1CPIJ*(WPIJ)**1.5)/(WPIJ+DELPIJ)**2.
      P3=.000452/(WPIJ+DELPIJ)
      P4=-(.000452*WPIJ-1.69*CPIJ*(WPIJ)**1.5)/(WPIJ+DELPIJ)**2.
      FL1=(PART2+P1*C(J,L)-P2*H(J,L)+P3*W(J,L)+P4*DEL(J,L))*DT
      FM1=0.
      FN1=0.
      H2=H(J,L)+FK1/2.
      C2=C(J,L)+FL1/2.
      W2=W(J,L)+FM1/2.
      DEL2=DEL(J,L)+FN1/2.
      FK2=(PART1-2.540*H2*(WPIJ)**.5)*DT
      FL2=(PART2+P1*C2-P2*H2+P3*W2+P4*DEL2)*DT
      FM2=0.
      FN2=0.
      H3=H(J,L)+FK2/2.
      C3=C(J,L)+FL2/2.
      W3=W(J,L)+FM2/2.
      DEL3=DEL(J,L)+FN2/2.
      FK3=(PART1-2.540*H3*(WPIJ)**.5)*DT
      FL3=(PART2+P1*C3-P2*H3+P3*W3+P4*DEL3)*DT
      FM3=0.
      FN3=0.
      H4=H(J,L)+FK3/2.
      C4=C(J,L)+FL3/2.
      W4=W(J,L)+FM3/2.
      DEL4=DEL(J,L)+FN3/2.
      FK4=(PART1-2.540*H4*(WPIJ)**.5)*DT
      FL4=(PART2+P1*C4-P2*H4+P3*W4+P4*DEL4)*DT
      FM4=0.
      FN4=0.
      DH=(FK1+2.*(FK2+FK3)+FK4)/6.
      DC=(FL1+2.*(FL2+FL3)+FL4)/6.
      DW=(FM1+2.*(FM2+FM3)+FM4)/6.
      DDEL=(FN1+2.*(FN2+FN3)+FN4)/6.
      H(J+1,L)=H(J,L)+DH
      C(J+1,L)=C(J,L)+DC
      W(J+1,L)=W(J,L)+DW
    5 DEL(J+1,L)=DEL(J,L)+DDEL
      DCBL=1.4
```

Table AVI - 2. (cont'd)

```
      CC(1,L)=C(1,L)
      CC(2,L)=C(21,L)
      CC(3,L)=C( 61,L)
      CC(4,L)=C(81,L)
      CC(5,L)=C(101,L)
      CC(6,L)=C(141,L)
      CC(7,L)=C(181,L)
    8 CC(8,L)=C(221,L)
      A1=0.
      A2=0.
      A3=0.
      A4=0.
      B1=0.
      B2=0.
      B3=0.
      B4=0.
      D1=0.
      D2=0.
      D3=0.
      D4=0.
      DO 9 M=1,MM
      A5(M)=CC(M,2)*CC(M,2)
    9 A1=A1+A5(M)
      DO 10 M=1,MM
      A6(M)=CC(M,3)*CC(M,2)
   10 A2=A2+A6(M)
      DO 11 M=1,MM
      A7(M)=CC(M,4)*CC(M,2)
   11 A3=A3+A7(M)
      DO 12 M=1,MM
      A8(M)=CC(M,2)*(CC(M,1)-CEXP(M))
   12 A4=A4+A8(M)
      DO 13 M=1,MM
      B5(M)=CC(M,2)*CC(M,3)
   13 B1=B1+B5(M)
      DO 14 M=1,MM
      B6(M)=CC(M,3)*CC(M,3)
   14 B2=B2+B6(M)
      DO 15 M=1,MM
      B7(M)=CC(M,4)*CC(M,3)
   15 B3=B3+B7(M)
      DO 16 M=1,MM
      B8(M)=CC(M,3)*(CC(M,1)-CEXP(M))
   16 B4=B4+B8(M)
      DO 17 M=1,MM
      D5(M)=CC(M,2)*CC(M,4)
   17 D1=D1+D5(M)
      DO 18 M=1,MM
      D6(M)=CC(M,3)*CC(M,4)
   18 D2=D2+D6(M)
```

Table AVI - 2. (cont'd)

```
      DO19M=1,MM
      D7(M)=CC(M,4)*CC(M,4)
   19 D3=D3+D7(M)
      DO20M=1,MM
      D8(M)=CC(M,4)*(CC(M,1)-CEXP(M))
   20 D4=D4+D8(M)
      AA=A1**B2*D3+B1*D2*A3+D1*B3*A2-A3*B2*D1-B3*D2*A1-B3*D1*A2
      AA1=A4**B2*D3+B4*D2*A3+D4*B3*A2-A3*B2*D4-B3*D2*A4-B3*B4*A2
      AA2=A1*B4*D3+B1*D4*A3+D1*B3*A4-A3*B4*D1-B3*D4*A1-B3*B1*A4
      AA3=A1*B2*B4+B1*D2*A4+D1*B4*A2-A4*B2*D1-B4*D2*A1-D4*B1*A2
      CODO=-AA1/AA
      WO=-AA2/AA
      DELO=-AA3/AA
      DO7J=1,JJ
      HSOL(J)=H(J,1)+CODO*H(J,2)+WO*H(J,3)+DELO*H(J,4)
      CSOL(J)=C(J,1)+CODO*C(J,2)+WO*C(J,3)+DELO*C(J,4)
      WSOL(J)=W(J,1)+CODO*W(J,2)+WO*W(J,3)+DELO*W(J,4)
    7 DELSOL(J)=DEL(J,1)+CODO*DEL(J,2)+WO*DEL(J,3)+DELO*DEL(J,4)
      DO31J=1,JJ
   31 IF(ABS(HSOL(J)-HP(J,1))-ERR1)32,32,40
   32 CONTINUE
      DO33J=1,JJ
   33 IF(ABS(CSOL(J)-CP(J,1))-ERR2)34,34,40
   34 CONTINUE
      DO35J=1,JJ
   35 IF(ABS(WSOL(J)-WP(J,1))-ERR3)36,36,40
   36 CONTINUE
      DO37J=1,JJ
   37 IF(ABS(DELSOL(J)-DELP(J,1))-ERR4)36,38,40
   38 CONTINUE
      GOTO42
   40 DO41J=1,JJ
      DO41L=1,4
      HP(J,L)=HSOL(J)
      CP(J,L)=CSOL(J)
      WP(J,L)=WSOL(J)
   41 DELP(J,L)=DELSOL(J)
      DO6J=1,JJ,2
      AJ=J
      T=(AJ-1.)*DT
    6 WRITE(3,103)K,J,T,CP(J,1),HP(J,1),WP(J,1),DELP(J,1)
      GOTO43
   42 AJ=J
      T=(AJ-1.)*DT
      WRITE(3,103)K,J,T,CP(J,1),HP(J,1),WP(J,1),DELP(J,1)
   43 STOP
      END
```

```
      DIMENSION HP(292,4),H(292,4),CP(292,4),C(292,4),WP(292,4,,W(292,4,
     1,DELP(292,4),DEL(292,4),HIP(4),CIP(4),WIP(4),DELIP(4,,CEXP(12),CC(
     212,4)
      DIMENSION DLTVX(4,5),S(7),DCVX(4,7)
      COMMON CC,CEXP,MM
  100 FORMAT(5I5)
  101 FORMAT(4F10.7)
  102 FORMAT(//,2X,10HITERATION=,I3,/)
  103 FORMAT(2I5,F8.1,4F12.6)
  104 FORMAT(1F10.3)
  105 FORMAT(2X,2HIT,3X,3HNO.,4X,2HT=,6X,4HCOD=,8X,5HRPIJ=,7X,2HW
     1=,9X,6HDELTA=,/)
  107 FORMAT(4F10.5)
  108 FORMAT(1F5.1,1F11.6)
  201 FORMAT(1)I5)
  202 FORMAT(7F10.0)
  203 FORMAT(//16H EVALUATION NO =I5/)
  204 FORMAT(6E13.6)
      READ(1,100)NN,KI,LL,JJ,MM
      WRITE(3,100)NN,KI,LL,JJ,MM
      DO11=1,4
      READ(1,101)HIP(I),CIP(I),WIP(I),DELIP(I)
    1 WRITE(3,101)HIP(I),CIP(I),WIP(I),DELIP(I)
      READ(1,101)HII,CII,WII,DELII
      WRITE(3,101)HII,CII,WII,DELII
      READ(1,108)DT,R
      WRITE(3,108)DT,R
      READ(1,107)G1,G2,G3,G4
      WRITE(3,107)G1,G2,G3,G4
      DO22M=1,MM
      READ(1,104)CEXP(M)
   22 WRITE(3,104)CEXP(M)
      DO2L=1,LL
      DO2J=1,JJ
      HP(J,L)=HII
      CP(J,L)=CII
      WP(J,L)=WII
    2 DELP(J,L)=DELII
      DO3I=1,4
      H(1,I)=HIP(I)
      C(1,I)=CIP(I)
      W(1,I)=WIP(I)
    3 DEL(1,I)=DELIP(I)
      READ(1,201)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      READ(1,202)ERROR,SUPLIM
      READ(1,202)(DCVX(I,1),I=1,NDIM)
      READ(1,202)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
      WRITE(3,201)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      WRITE(3,204)ERROR,SUPLIM
      WRITE(3,204)(DCVX(I,1),I=1,NDIM)
      WRITE(3,204)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
      DO6K=1,KI
      WRITE(3,102)K
      DO5L=1,LL
```

Table AVI - 3. (cont'd)

```
      DO5J=1,NN
      HPIJ=HP(J,L)
      CPIJ=CP(J,L)
      WPIJ=WP(J,L)
      DELPIJ=DELP(J,L)
      IF(L.GE.2)GO TO33
      PART1=R+.845*(HPIJ)**1.5
      PART2=(.845*CPIJ*(HPIJ)**1.5+.000452*WPIJ)/(HPIJ+DELPIJ)
      GOTO4
   33 PART1=0.
      PART2=0.
    4 FK1=(PART1-2.540*H(J,L)*(HPIJ)**.5)*DT
      P1=(-1.69*(HPIJ)**1.5)/(HPIJ+DELPIJ)
      P2=(2.54*CPIJ*(HPIJ)**.5)/(HPIJ+DELPIJ)+(.000452*WPIJ-1.69*
     1CPIJ*(HPIJ)**1.5)/(HPIJ+DELPIJ)**2.
      P3=.000452/(HPIJ+DELPIJ)
      P4=-(.000452*WPIJ-1.69*CPIJ*(HPIJ)**1.5)/(HPIJ+DELPIJ)**2.
      FL1=(PART2+P1*C(J,L)-P2*H(J,L)+P3*W(J,L)+P4*DEL(J,L))*DT
      FM1=0.
      FN1=0.
      H2=H(J,L)+FK1/2.
      C2=C(J,L)+FL1/2.
      W2=W(J,L)+FM1/2.
      DEL2=DEL(J,L)+FN1/2.
      FK2=(PART1-2.540*H2*(HPIJ)**.5)*DT
      FL2=(PART2+P1*C2-P2*H2+P3*W2+P4*DEL2)*DT
      FM2=0.
      FN2=0.
      H3=H(J,L)+FK2/2.
      C3=C(J,L)+FL2/2.
      W3=W(J,L)+FM2/2.
      DEL3=DEL(J,L)+FN2/2.
      FK3=(PART1-2.540*H3*(HPIJ)**.5)*DT
      FL3=(PART2+P1*C3-P2*H3+P3*W3+P4*DEL3)*DT
      FM3=0.
      FN3=0.
      H4=H(J,L)+FK3/2.
      C4=C(J,L)+FL3/2.
      W4=W(J,L)+FM3/2.
      DEL4=DEL(J,L)+FN3/2.
      FK4=(PART1-2.540*H4*(HPIJ)**.5)*DT
      FL4=(PART2+P1*C4-P2*H4+P3*W4+P4*DEL4)*DT
      FM4=0.
      FN4=0.
      DH=(FK1+2.*(FK2+FK3)+FK4)/6.
      DC=(FL1+2.*(FL2+FL3)+FL4)/6.
      DW=(FM1+2.*(FM2+FM3)+FM4)/6.
      DDEL=(FN1+ 2.*(FN2+FN3)+FN4)/6.
      H(J+1,L)=H(J,L)+DH
      C(J+1,L)=C(J,L)+DC
      W(J+1,L)=W(J,L)+DW
    5 DEL(J+1,L)=DEL(J,L)+DDEL
      DO8L=1,4
      CC(1,L)=C(1,L)
      CC(2,L)=C(21,L)
```

Table AVI - 3. (cont'd)

```
      CC(3,L)=C( 81,L)
      CC(4,L)=C(81,L)
      CC(5,L)=C(101,L)
      CC(6,L)=C(141,L)
      CC(7,L)=C(181,L)
    8 CC(8,L)=C(221,L)
      CALL GKCHEN(NDIM,METHOD,MAXNO,ERROR,SUPLIM,DLTVX,DCVX,S,KK)
      WRITE(3,204)S(NDIM+2),(DCVX(I,NDIM+2),I=1,NDIM)
      WRITE(3,204)((DCVX(I,J),I=1,NDIM),J=1,NOPT)
      WRITE(3,204)(S(I),I=1,NOPT)
      WRITE(3,203)KK
      WRITE(3,105)
      DO7J=1,JJ
      HSOL=H(J,1)+DCVX(1,1)*H(J,2)+DCVX(2,1)*H(J,3)+DCVX(3,1)*H(J,4)
      CSOL=C(J,1)+DCVX(1,1)*C(J,2)+DCVX(2,1)*C(J,3)+DCVX(3,1)*C(J,4)
      WSOL=W(J,1)+DCVX(1,1)*W(J,2)+DCVX(2,1)*W(J,3)+DCVX(3,1)*W(J,4)
      DELSOL=DEL(J,1)+DCVX(1,1)*DEL(J,2)+DCVX(2,1)*DEL(J,3)+DCVX(3,1)*DE
     1L(J,4)
      DO7L=1,4
      HP(J,L)=HP(J,L)+G1*(HSOL-HP(J,L))
      CP(J,L)=CP(J,L)+G2*(CSOL-CP(J,L))
      WP(J,L)=WP(J,L)+G3*(WSOL-WP(J,L))
    7 DELP(J,L)=DELP(J,L)+G4*(DELSOL-DELP(J,L))
      DO6J=1,JJ,4
      AJ=J
      T=(AJ-1.)*DT
    6 WRITE(3,103)K,J,T,CP(J,1),HP(J,1),WP(J,1),DELP(J,1)
      RETURN
      END
```

Table AVI - 3. (cont'd)

```
      SUBROUTINE GKCHEN(NDIM,METHOD,MAXNO,ERROR,SUPLIM,DLTVX,DCVX,S,KK)
      DIMENSION DLTVX(4,5),C(5),DCVX(4,7),S(7),CNTROX(4)
  110 FORMAT(/19H THIS IS NEW METHOD/)
  111 FORMAT(/16H THIS IS SIMPLEX/)
  112 FORMAT(/12H THIS IS BOX/)
  113 FORMAT(/16H ****WARNING****/)
  114 FORMAT(  50H INADEQUATE GIVEN MAX. NO FOR FUNCTION EVALUATION,)
  115 FORMAT(47H INCREASING THE MAXNO OR CHANGING THE STEP SIZE/)
      GO TO (116,117,118),METHOD
  116 JMCHEN=1
      KCHEN=1
      ALPHO=1.0
      BETA=0.5
      COEFF=1.2
      GAMMA=2.0
      WRITE(3,110)
      GO TO 1
  117 JMCHEN=1
      KCHEN=2
      ALPHO=1.0
      BETA=0.5
      GAMMA=2.0
      WRITE(3,111)
      GO TO 1
  118 JMCHEN=NDIM
      ALPHO=1.3
      BETA=0.5
      WRITE(3,112)
    1 J=1
      KK=1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      K=NDIM+JMCHEN
      KLT1=K-1
      DO 3  J=2,K
      DO 2 I=1,NDIM
    2 DCVX(I,J)=DCVX(I,1)+DLTVX(I,J-1)
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
    3 CONTINUE
    4 M=K
      ALPHA=ALPHO
      CALL ORDER(M,NDIM,S,DCVX)
      DO 5  I=1,KLT1
    5 C(I)=1.
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
    6 DO 7  I=1,NDIM
```

Table XVI - 3. (cont'd)

```
 7 DCVX(I,K+1)=CNTROX(I)+ALPHA*(CNTROX(I)-DCVX(I,K))
   J=K+1
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
   IF(KK-MAXNO)8,8,36
 8 GO TO (9,9,23),METHOD
 9 IF(S(K+1)-S(1))10,10,23
10 DO 11 I=1,NDIM
11 DCVX(I,K+2)=CNTROX(I)+GAMMA*(DCVX(I,K+1)-CNTROX(I))
   J=K+2
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
   IF(KK-MAXNO)12,12,36
12 GO TO (16,13),KCHEN
13 IF(S(K+2)-S(1))14,14,21
14 S(K)=S(K+2)
   DO 15 L=1,NDIM
15 DCVX(L,K)=DCVX(L,K+2)
   GO TO 35
16 IF(S(K+2)-S(K+1))17,17,21
17 S(K)=S(K+2)
   DO 18 L=1,NDIM
18 DCVX(L,K)=DCVX(L,K+2)
   M=K
   CALL ORDER(M,NDIM,S,DCVX)
   CALL SCHECK(K,SUM,NDIM,S)
   IF(SUM-ERROR)37,37,19
19 CVALUE=2*NDIM-1
   DO 20 I=1,KLT1
   C(I)=CVALUE
20 CVALUE=2*NDIM-2
   CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
   ALPHA=ALPHO*COEFF
   GO TO 6
21 S(K)=S(K+1)
   DO 22 L=1,NDIM
22 DCVX(L,K)=DCVX(L,K+1)
   GO TO 35
23 IF(S(K+1)-S(K-1))21,21,24
24 IF(S(K+1)-S(K))25,25,27
25 S(K)=S(K+1)
   DO 26 I=1,NDIM
26 DCVX(I,K)=DCVX(I,K+1)
27 DO 28 I=1,NDIM
28 DCVX(I,K+1)=CNTROX(I)+BETA*(DCVX(I,K)-CNTROX(I))
   J=K+1
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
   IF(KK-MAXNO)29,29,36
29 IF(S(K+1)-S(K))30,30,32
30 S(K)=S(K+1)
   DO 31 I=1,NDIM
31 DCVX(I,K)=DCVX(I,K+1)
   GO TO 35
```

Table AVI - 3. (cont'd)

```
   DO 34 J=2,K
   DO 33 I=1,NDIM
33 DCVX(I,J)=(DCVX(I,1)+DCVX(I,J))/2.
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
34 CONTINUE
   IF(KK-MAXNO)35,35,36
35 CALL SCHECK(K,SUM,NDIM,S)
   IF(SUM-ERROR)37,37,4
36 WRITE(3,113)
   WRITE(3,114)
   WRITE(3,115)
   GO TO 40
37 DO 38 I=1,KLT1
38 C(I)=1.
   CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
   DO 39 I=1,NDIM
39 DCVX(I,K+1)=CNTROX(I)
   J=K+1
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
40 RETURN
   END
```

Table AVI - 3. (cont'd)

```
      SUBROUTINE SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      DIMENSION S(7),DCVX(4,7),X(3),XOPT(4),AII(12),AI(12),CC(12,4),CEXP
     1(12)
      COMMON CC,CEXP,MM
    1 FORMAT(31H THE OPTIMUM FUNCTION VALUE IS E13.6)
    2 FORMAT(6E13.6)
    3 FORMAT(10I4)
      IF(J-1)4,4,5
    4 KCONT=10
      ERR=10.
      GO TO 6
    5 KK=KK+1
    6 DO 7  I=1,NDIM
      X(I)=DCVX(I,J)
    7 CONTINUE
      T=0.
      DO99I=1,MM
      AII(I)= CC(I,1)+X(1)*CC(I,2)+X(2)*CC(I,3)+X(3)*CC(I,4)-CEXP(I)
      AI(I)=AII(I)*AII(I)
   99 T=T+AI(I)
      S(J)=T
      IF(J-1)9,9,11
    9 DO 10 I=1,NDIM
      XOPT(I)=X(I)
   10 CONTINUE
      SOPT=T
      IF(J-1)17,17,12
   11 IF(S(1)-S(J))12,9,9
   12 IF(KK-KCONT)14,13,13
   13 WRITE(3,1)SOPT
      WRITE(3,2)(XOPT(I),I=1,NDIM)
      WRITE(3,3)KK
      KCONT=KCONT+10
   14 IF(S(J)-ERR)15,15,17
   15 WRITE(3,1)SOPT
      WRITE(3,2)(XOPT(I),I=1,NDIM)
      WRITE(3,3)KK
      ERR=ERR*0.1
      GO TO 17
   16 S(J)=SUPLIM
   17 RETURN
      END
```

Table AVI - 3. (cont'd)

```
      SUBROUTINE ORDER(M,NDIM,S,DCVX)
      DIMENSION S(7),DCVX(4,7)
      K=M
      KLT1=K-1
      DO  5 I=1,KLT1
      M=M-1
      DO  4 J=1,M
      IF(S(M+1)-S(J))2,2,4
    2 A=S(M+1)
      S(M+1)=S(J)
      S(J)=A
      DO  3 L=1,NDIM
      B=DCVX(L,M+1)
      DCVX(L,M+1)=DCVX(L,J)
      DCVX(L,J)=B
    3 CONTINUE
    4 CONTINUE
    5 CONTINUE
      RETURN
      END
```

Table AVI - 3. (cont'd)

```
      SUBROUTINE CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      DIMENSION C(5),CNTROX(4),DCVX(4,7)
      CSUM=0.
      DO 1  I=1,KLT1
    1 CSUM=CSUM+C(I)
      DO 3  I=1,NDIM
      AXIS=0.
      DO 2  J=1,KLT1
      CNTROX(I)=AXIS+C(J)*DCVX(I,J)
      AXIS=CNTROX(I)
    2 CONTINUE
      CNTROX(I)=CNTROX(I)/CSUM
    3 CONTINUE
      RETURN
      END
      SUBROUTINE SCHECK(K,SUM,NDIM,S)
      DIMENSION S(7)
      SAVG=0.
      DO 1  L=1,K
    1 SAVG=S(L)+SAVG
      AK=K
      SAVG=SAVG/AK
      SUM=0.
      DO2 L=1,K
    2 SUM=SUM+(S(L)-SAVG)**2
      ANDIM=NDIM
      SUM=SUM **0.5/ANDIM
      RETURN
      END
```

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NOMENCLATURE

a	parameter of a system
$a(0)$	initial condition of a
$a_{j,n+1}$	integration constants
a^*	a constant
A	area of feedlot
A_i $(i=1, \ldots 4)$	functionals defined by Equations (144) through (147)
b	amplitude of variation
b_i $(i=1, 2, 3)$	amplitude of variation
B_i $(i=1,2,..4)$	functionals defined by Equations (148) through (151)
c	dimensionless concentration of COD
C	concentration of COD
C_o	initial concentration of COD
$C_e(t_i)$	experimental data of COD concentration at $t = t_i$
C_{n+1}	complete solution of COD for (n+1)st iteration
$C^h_{j,n+1}$	jth homogeneous solution of COD concentration
C^p_{n+1}	particular solution of COD concentration
C^*	a characteristic COD concentration
D_i $(i=1, .. 4)$	functionals defined by Equations (152) through (155)
f	forcing function of a system
H	water head over weir crest
H_{n+1}	complete solution of water head for (n+1)st iteration
$H^h_{j,n+1}$	jth homogeneous solution of water head

H^p_{n+1}	particular solution of water head
i	a constant
j	a constant
k_1, k_2, k_3	constants
K_1, K_2, K_3	constants
L	width of feedlot system
m	a constant
n	a constant
q	dimensionless outflow rate
Q	out flow rate
Q_{n+1}	complete solution of outflow rate for (n+1)st iteration
$Q^h_{j,n+1}$	jth homogeneous solution of outflow rate
Q^p_{n+1}	particular solution of outflow rate
r	dimensionless rainfall intensity
r_s	steady state component of dimensionless rainfall intensity
R	rainfall intensity
R_s	average rainfall intensity
t	time
T	characteristic time of the system
v	dimensionless tank volume
V	tank volume
W	injection rate of COD
W_{n+1}	complete solution of injection rate for (n+1)st iteration
$W^h_{j,n+1}$	jth homogeneous solution of injection rate
W^p_{n+1}	particular solution of injection rate
x	independent variable of a system

y	state of the system
$y^h_{j,n+1}$	jth homogeneous solution of y
y^p_{n+1}	particular solution of y
y^*	constant
$y_e(x_i)$	observation on y at $x = x_i$

Greek Letters

α	dimensionless parameter of the system
β	dimensionless parameter of the system
γ	dimensionless parameter of the system
δ	weir height
θ	dimensionless time
λ	a constant
ξ_i	constants between 0 and 1
Φ	minimizing functional in curve fitting
ω	angular frequency
σ	standard deviation of a normal distribution
μ	mean of a normal distribution

MATHEMATICAL MODELING AND SYSTEM ANALYSIS OF CATTLE FEEDLOT RUNOFF

by

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B. S., National Taiwan University, 1965

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

The cattle industry has undergone radical changes within the past decade. The water pollution aspects of cattle feedlots have become of wide concern in the midwest. In the past, however, little attention was given to manure disposal and pollution control in the design of animal feeding facilities.

The nature of runoff from cattle feedlots has been studied by Miner (1) in both quantitative and qualitative aspects. The considerations of the dynamic characteristics of the system's behavior are, however, relatively untouched. Independent variables of the system as a function of time are important in the case of extended rainfall events. This study emphasizes the dynamic characteristics of the system's behavior. The experimental data obtained by Miner are used as source data in this study.

A simplified nonlinear model of the system is first proposed. An analog computer is used to solve the nonlinear equations. System parameters and missing initial conditions are recovered by fitting analog solutions to the experimental data. Correlation of the system parameter to the rainfall intensity is discussed. Dimensional analysis is then employed to obtain the characteristic dimensionless groups which may be used to investigate the system's behavior more generally. The unsteady state property of rainfall as a function of time is also taken into account by the computer simulation technique. The sinusoidal, square-wave and random functional responses are thus observed. Finally, a general technique for parameter determination and problem identification is discussed, namely the quasilinearization technique. Use of this technique is effective when the number of system parameters

is increased, and convergence exists.

Analog computer simulation was found to be satisfactory in the study of transient behavior of the system. Based on the proposed model, the injection rate of the organic matter into the runoff solution is found to be very approximately linear with respect to the rainfall intensity. Strangely enough, this injection rate is somewhat independent of the surface condition of the feedlot system. Constant rainfall intensity is a good approximation according to the simulation study by assuming various time-variant rainfall functions. The scattering of the experimental data limits the extensive use of the quasilinearization technique. Convergence could not be achieved for several sets of data.

For a complex physical system, the response of the system under varied input variables may be the most informative knowledge to justify a proposed modeling. It is suggested that additional hydraulic and concentration data be taken systematically and in a shorter time interval.